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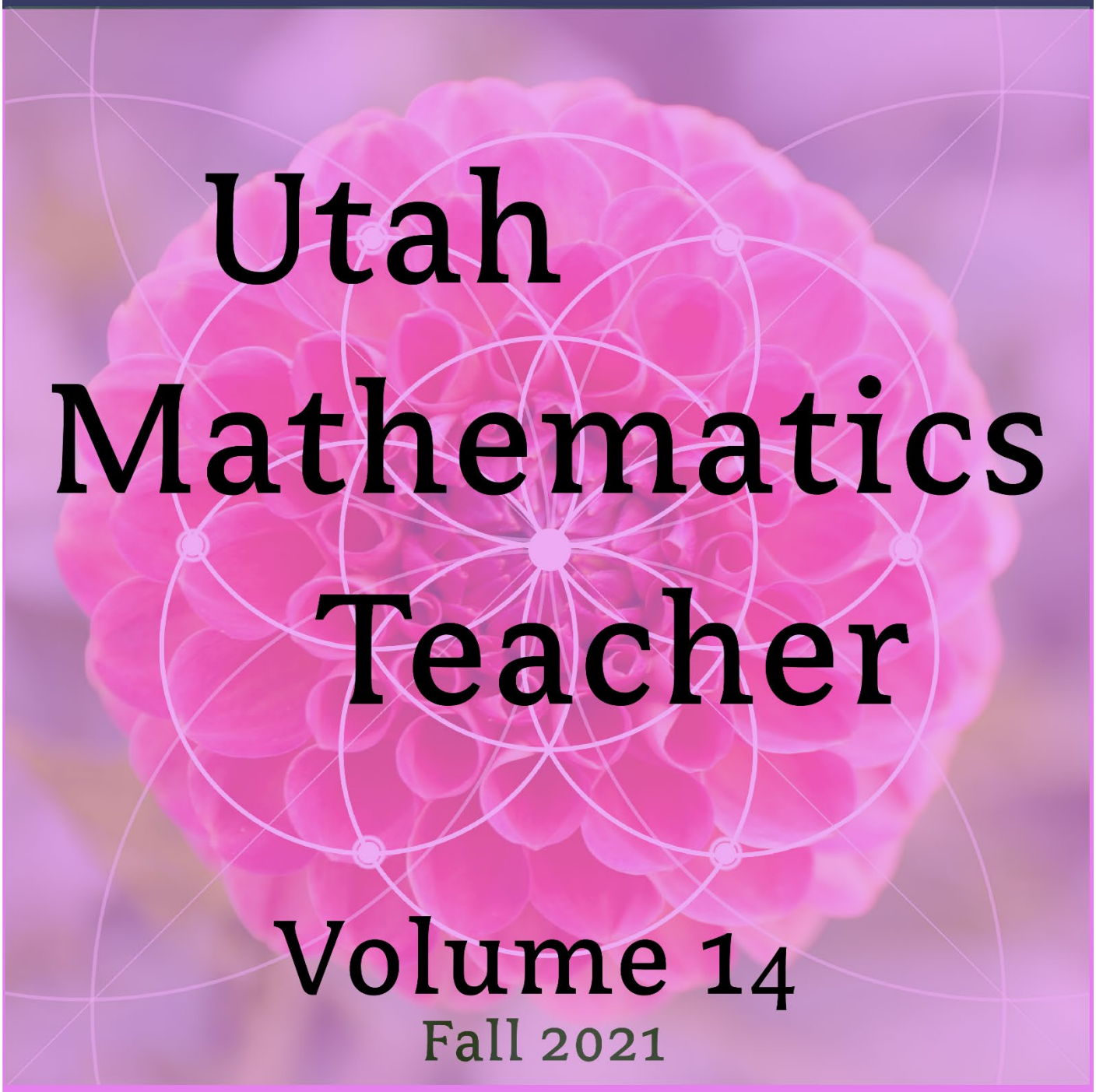

Utah Mathematics Teacher

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"Making Mathematics Flourish"

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The Utah Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Danielle Divis at danielledivis21@gmail.com. A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included.

UTAH MATHEMATICS TEACHER

Volume 14, Fall 2021

"Making Mathematics Flourish"

Table of Contents

UCTM President's Message

Andrew Glaze, Ph.D.

There is a trending mantra I see on classroom walls and doors. It reads "How to be a math person: Step 1: Do Math (Any type). Step 2: Be a person." What a wonderfully inclusive way to think about our mathematics learners. That sign probably resonates with me because I am among the myriad of people who internalized the "I'm not a math person" mantra throughout my k-12 experience. I even had the grades and pathway to prove it. My high school geometry teacher validated my feelings when he declared that I would never be very good at math on my scholarship recommendation (I didn't get the scholarship, by the way!).

I got lucky. I was eventually able to find and foster a love for mathematics in college. My major required me to take college calculus. My perspective on mathematics changed when I successfully completed that course. Calculus was both complicated and fascinating. My success in Dr. Orr's calculus course propelled me forward to tackle other complicated mathematical topics and to eventually find a fulfilling career sharing mathematics with others. My life would be completely different now if I never had that mathematical experience. As mathematics teachers we have the privilege of being "math people" advocates. We can help others see the beauty of mathematics. It is my hope that we will continually share the message that we are all math people. We can help spread the message that we are all math people by giving the underrepresented a voice in our classrooms. We can help spread the message that we are all math people by helping others understand that mathematics is more than a set of procedures, a correct numerical answer, or a collection of state standards. We can spread the message that we are all math people by providing empowering and challenging mathematics in our classrooms. How will you spread the message that we are all math people?

Letter from the Editor

Danielle Divis, RSL Academy High School

In light of our upcoming conference in 2022, we really wanted to highlight our UCTM theme of "Making Math Flourish" in our journal article selections for this year. I'm personally passionate about these theme. As a doctoral student, I have had the opportunity this year to study students working on mathematics that was contextualized with music theory. It was such a pleasure to watch their mathematical dispositions improve and the interest piqued while doing mathematics in connection to music. This example illustrates just one of the many ways that we as mathematics educators can and have a responsibility to make mathematics flourish for and within our students. These articles each contribute to our understanding of how this can be done in the classroom. I'd like to highlight some of our feature articles.

In "*From Playful Math Explorations to Beautiful Origami*," Violeta Vasilevska does an excellent job connecting to our theme this year by illustrating how origami activities connect to mathematical principles across all grade levels, including post secondary! Next in our journal, Sydney Tesch and Lindsey Henderson help us understand how data science problems can help students flourish through their development of critical thinking skills. Finally, a thorough research study by Shannon Olson demonstrates the importance of professional learning in mathematics teacher satisfaction and retention. It is only through our continued effort to improve our teaching and teacher training, using the suggestions like these presented in this journal by wonderful Utah educators across the state, that we are able to help our Utah students flourish in their mathematics classrooms.

I hope you enjoy this journal as much as I enjoyed collecting, reading, reviewing, and discussing the articles with the review committee. In addition, as always, please consider submitting your own articles, or serving as a reviewer for future journal articles.

Note: Any mistakes are the sole responsibility of the editor and will be remedied in the online journal. Please send corrections to Danielledivis21@gmail.com.

From Playful Math Explorations to Beautiful Origami Creations

Violeta Vasilevska (Utah Valley University)

Abstract

Origami has been fascinating and drawing people into practicing this amazing paper folding art for centuries. It's simple and yet challenging folds resulting in marvelous creations has been enchanting the world we know. What is even more wonderous about origami is the fact that every single fold is guided by mathematical properties and laws. This article discusses a few project ideas that connect origami with math that have been used with middle school, high school, as well as college students such as: introducing origami geometry and its connection with geometry constructions and using math to make a 3-colored origami dodecahedron. In addition, following Francis Su's invitation to use math for human flourishing, some of the desires and virtues that are nurtured throughout these projects are discussed.

Introduction

Francis Su's MAA President Retiring Address (Su, 2017) started something that has been waiting to nourish in each of us: the need to talk about and become better human beings through imagining new ways of mathematics for human flourishing.

In his book *Mathematics for Human Flourishing*, Francis Su (2020) gave us a powerful and beautiful narrative and reflections on some basic human desires (such as for play, exploration, beauty, struggle, love, etc.) on which mathematics is based, and the accompanying virtues it cultivates that are essential for human flourishing. How can these desires lead us through mathematics to make us and our students flourish as human beings? How can we make our students see the beauty in mathematics? How can we nourish the love of mathematics for them? How can we make them see the playfulness of mathematics in every simple thing they are doing daily?

In this article, I will elaborate on some of the project ideas I have frequently used in my classes and during many high school outreach programs I have participated in. Those discovery projects reveal various connections of math with origami through play, exploration, solving puzzles, and hands-on experiences. These projects provide opportunities for students to see the playfulness and beauty of mathematics that is lurking behind astonishing origami creations.

Origami & Math - Meaningful Connections

Robert Lang (a professional origami artist and a physicist) in his *Wired* talk explains the deep connections between math and origami that I was not aware of until 15 years ago:

> …I use a lot of math in my work [with origami]…And that's because what's possible in origami is defined by the mathematical properties of the folded sheet of paper. So, if you understand the math, you can use it to create a lot of forms that you probably wouldn't have discovered just by trial and error. (Lang, 2008)

I have been attracted to crafty art and the challenge of doing it for as long as I can remember. I have always loved doing Origami (Japanese word for "paper folding:" ori = "folded" and kami = "paper") and enjoyed and admired the beautiful creations that were the result of a sometimes frustrating but nevertheless rewarding process. Sometimes the struggle during this process was overwhelming, sometimes it took lots of trials to get the right fold, and sometimes the challenge was enormous. However, as time was passing and I was getting familiar with the folds, the folding was getting easier, more relaxing, and definitely more enjoyable. Moreover, with time the challenges seemed to be just small hills, not mountains to overcome and prompted me to explore more folding techniques. And every time, the final results led to creating something that I was proud to show to my family and friends.

Does this remind us of something? Don't we go through the same processes when we are doing mathematics? Most of the time we struggle while doing mathematics, we are challenged to the point of quitting, but as time passes and with practice, creativity, patience, and perseverance we overcome these obstacles and become better and better in what we are doing. Those are some of the virtues that Su (2020) addressed in his book and I have cultivated while pursuing mathematics – they have definitely helped me in enjoying and becoming better in my paperfolding.

In addition, for years, I practiced origami exclusively as a hobby and never really thought about any connections with mathematics. When in 2005 I read about a workshop on *Origami in Undergraduate Mathematics Courses* offered by Thomas Hull at the Joint Mathematics Meetings – I was very skeptical. I had done origami for so long and had done math all my life – what could possibly be the connection between the two? My curiosity and love for learning led me to take the workshop. That was one of my best decisions and a pivotal moment in realizing how much math is behind every single origami fold.

Showing students those connections between mathematics and everyday tasks and nurturing these desires and virtues, is something that we all need to do more for our own benefit as well as for our students' well-being.

In the next two sections, I will present some powerful mathematics ideas that are behind the origami folds and some beautiful origami creations.

Origami & Math – Beautiful and Playful Connections

Wonder, Explore, Discover

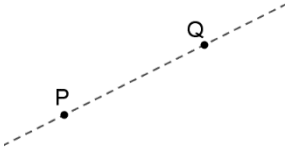
In their geometry classes, high school students are exposed to and learn about geometric constructions using only a straight edge and a compass (SE&C): drawing a line through two points, bisecting a segment and angle, drawing perpendicular lines, etc. All these constructions can be accomplished by four basic axioms (first defined by Euclid) (Venema, 2002). The fascinating discovery for me was that all these geometric SE&C constructions are closely related to origami.

Let's look at a square sheet of origami paper. Origami constructions are folding constructions that can be accomplished by folding the origami paper using the original edges and

vertices of the paper, as well as any crease lines or points obtained while folding. Using the side of the paper as the straight edge and folding up the side to an angle to simulate a compass, we can make geometric constructions with origami. The Italian-Japanese mathematician Humiaki Huzita has formulated a set of six origami axioms as a base for origami geometry (Huzita, 1992). What is interesting about the first five origami axioms is that everything that can be constructed by a SE&C can be folded by origami using only those five axioms (Hull, 2006; Hvidsen, 2005).

The illustrations below provide connections of some basic origami folds with the corresponding origami axioms and the equivalent SE&C constructions (Hvidsen, 2005). Note that a fold line created by an axiom (during that step) is indicated by a dotted line, and the direction of the fold is indicated by a curved arrow. The solid lines are the sides of the square or are lines already constructed in previous steps.

Origami Axiom 1 (OA1)
Given two constructed points P and Q, we can construct a line (fold) through them.



Equivalent SE&C construction:
A line joining two points.

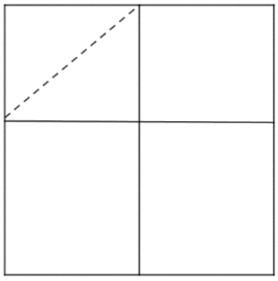
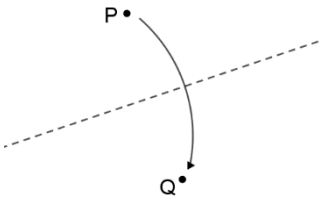


Figure 1: Basic origami fold (OA1)

Origami Axiom 2 (OA2)
Given two constructed points P and Q, we can fold P onto Q.



Equivalent SE&C construction:
The perpendicular bisector of a segment.

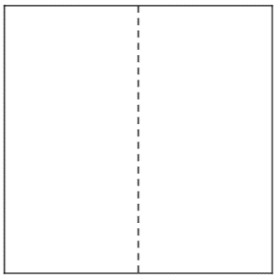
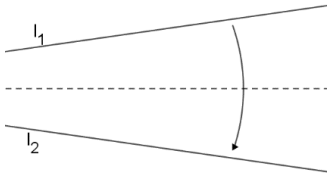


Figure 2: Basic origami fold (OA2)

Origami Axiom 3 (OA3)
Given two constructed lines l_1 and l_2, we can fold line l_1 onto l_2.



Equivalent SE&C construction:
The angle bisector of the angle formed by two intersecting lines.

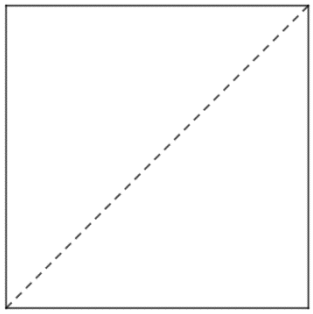
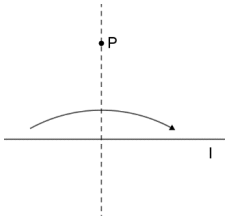


Figure 3: Basic origami fold (OA3)

Origami Axiom 4 (OA4)
Given a constructed point P and a constructed line l, we can construct a perpendicular to l passing through P.



Equivalent SE&C construction:
The perpendicular to a line through a point.

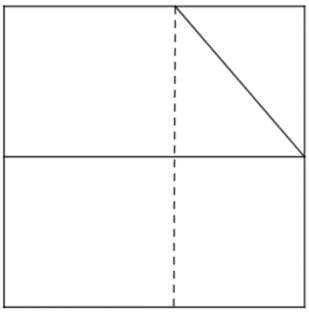
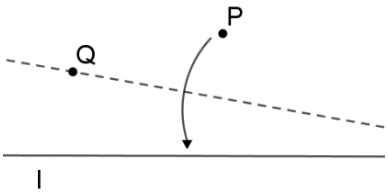


Figure 4: Basic origami fold (OA4)

Origami Axiom 5 (OA5)
Given two constructed points P and Q and a constructed line l, then whenever possible, the line through Q, which reflects P onto l, can be constructed.



Equivalent SE&C construction:
Tangent to a parabola with focus P and directrix l.

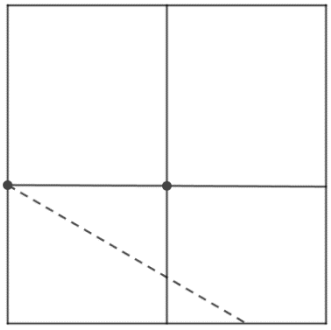
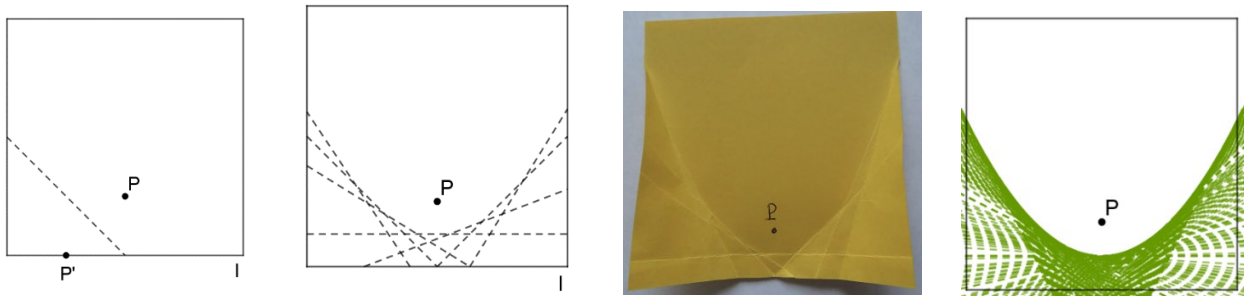


Figure 5: Basic origami fold (OA5)

Students usually can easily discover the connections of the origami creases (folds) obtained by OA1-OA4 with the associated SE&C constructions. However, they are not able to see nor guess easily what the crease created by OA5 represents. What is surprising for students and a big WOW moment for them, is the result of the following exercise (Hull, 2006) credited to Rupp (1924): choose a point P somewhere on the origami square paper and fold the point P onto one of the sides of the paper (say line l), by bending the paper until l touches P – then flatten the crease (Figure 6a). This is asking students to apply OA5. Students can be guided in discovery that the crease is a perpendicular bisector of the segment PP'. The actual surprise reveals when students are asked to repeatedly apply OA5 – they now see that the constructed tangents outline a parabola with the focus P and a directrix l (Figure 6b). The Figure 6c and 6d show the resulting origami paper outcome and the GeoGebra illustration of applying OA5 over and over.



a) b) c) d)

Figure 6: Activity 4: Folding a parabola (Hull, 2006)

It is worth noting that applying OA5 followed by OA4 constructs a point on a parabola with focus *P* and directrix *l* (4.4. Mini-Project – Origami Constructions (Hvidsen, 2005); Activity 6: Folding a Parabola (Hull, 2006)).

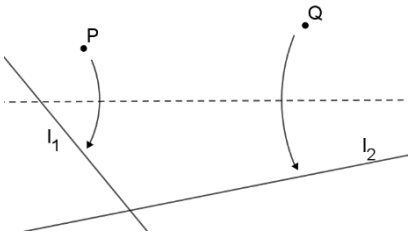
To illustrate how these origami axioms are used in folding fascinating origami creations, one can provide instructions of making an origami box with accompanying commentary on the origami axioms used for each constructed crease (fold). Note, usually OA1-3 are used for construction of an easy origami box (Masu Box (Beech, 2002, pp. 60-61))[1]. This easy project allows students to not just fold the box and admire their creation, but also identify and discuss the origami axioms (as well as the accompanying SE&C constructions) they are using to fold the final model.

Above folds illustrated how some SE&C constructions can be done using the five Huzita's origami axioms. However, it is well known that certain geometric constructions are impossible to do with an SE&C, like trisecting an arbitrary angle, or doubling the volume of a cube (i.e., constructing the cube root of 2). This is where the fascination with origami happens: origami constructions are more powerful than SE&C constructions. Namely, trisecting angles and doubling cubes **can be accomplished** with origami constructions (Activity 5: Can Origami Trisect an Angle

[1] Origami box instructions can be easily found through an internet search.

(Hull, 2006)). The method of trisecting an angle using origami in Hull's book is due to H. Abe (Hushimi, 1980) and the method of doubling the cube using origami was developed by Peter Messer (1986). Both methods rely on and can be achieved by Huzita's origami axiom 6.

Origami Axiom 6 (OA6)
Given two constructed points P and Q and two constructed lines l_1 and l_2, then whenever possible, a line that reflects P onto l_1 and also reflects Q onto l_2 can be constructed.



No equivalent SE&C construction.

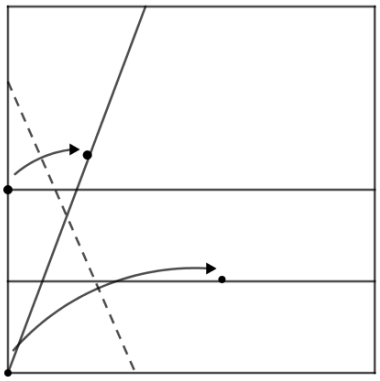


Figure 6: Basic origami fold (OA6)

OA6 constructs a line that is tangent to two parabolas drawn in the plane. Hence, OA6 can solve cubic equations, something that SE&C cannot do.

The even more surprising and fascinating aspect of the Huzita's axioms is the fact that they have been used in real-life applications as well. Robert Lang's quote and his short *Wired* video (Lang, 2008) give just a glimpse of how powerful these axioms are (Lang, 2010). In addition, origami folds have been extensively used in medicine, engineering, space programs, robotics, architecture, etc. (Origami Resource Center, 2021). Students as well as teachers are always enchanted by these powerful applications of the origami folds and the math behind them.

The Math Behind Stunning Modular Origami

Imagination, Creativity, Curiosity, Patience, Perseverance

For decades, mathematicians have been intrigued by and interested in the aspect of mathematical origami as a puzzle. They have been competing in designing realistic or complex origami models using only one sheet of square paper to fold into a model, without cutting (Lang, 2021). What is interesting is that mathematicians are interested not just in the final shape of the model, but also in the crease pattern (an origami diagram that contains (almost) all of the creases in the final model) as well as the sequence of folds. In addition, they are also interested in modular origami because of the challenge that it offers as a puzzle. *Modular origami* is a paper folding technique that uses more than one sheet of paper and consists of two steps

1. folding modules (units) from each individual sheet of paper (usually an easy step); and then

2. putting these units together to create a larger and more complex origami creation (Origami Resource Center – Modular, 2021). This step is usually a challenging one and involves solving various tasks (puzzles).

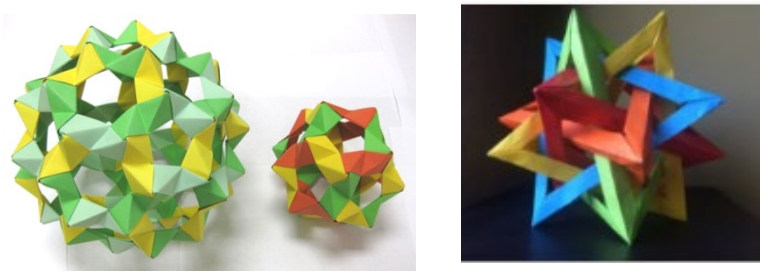


Figure 7: Modular origami

In this section our discussion will be on solving the following puzzle:

How to make an origami dodecahedron (Figure 8) such that the edges coming out of a vertex are colored with three different colors?

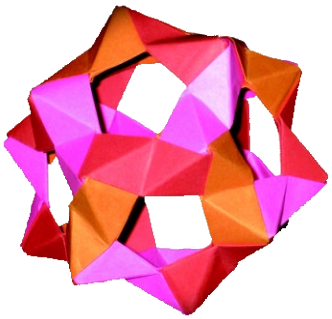


Figure 8: 3-colored origami dodecahedron

The origami dodecahedron shown on Figure 8 is a modular origami. The units used to construct this origami model are called PHiZZ units (Pentagon-Hexagon Zig-Zag units). They were invented by Thomas Hull in 1993. The name indicates that they can be used to make any polyhedron that is cubic – each vertex has three edges meeting it and has only pentagon and hexagon faces. Making these units is time consuming, but otherwise it is a relatively easy task (Hull, 2000). One can try to assemble the PHiZZ units and make a 3-colored origami dodecahedron by trial and error and eventually they might be able to make this beautiful 3-colored origami dodecahedron.

However, the questions we want to address are: Can we solve this puzzle – making the 3-colored origami dodecahedron – without trial and error? Can we use math to achieve this goal? If so, how?

The ideas presented in this section to answer those questions are drawn from Activity 12: Making Origami Buckyballs in Hull's book (2006).

High school students are familiar with representing polyhedra (3-dimensional objects) with *net* – 2-dimensional representation. However, there is a different 2-dimensional representation of a

polyhedron that is not discussed during high school. This representation can be obtained using planar graphs. Given a polyhedron, a *planar graph* of a polyhedron is a 2-dimenional representation of the polyhedron that can be drawn on the plane as follows:

- represent the vertices of the polyhedron with dots (called *graph vertices*) and
- represent each edge of the polyhedron with a line segment (called a *graph edge*) in such a way that
    - the line segment connects the dots corresponding to the edge vertices and
    - the line segments intersect only at their endpoints.

To draw a planar graph, a 2-dimensional representation of a polygon, imagine that the skeleton of the polygon is made of a stretchy material like rubber; put the polygonal skeleton on a flat surface; stretch the top face of the polygon; then push the top polygon face down on the flat surface making sure that none of the edges cross.

Asking students to use this method to draw planar graphs of a cube and a dodecahedron, gives them opportunity for visualization as well as creativity (Figures 9 and 10).

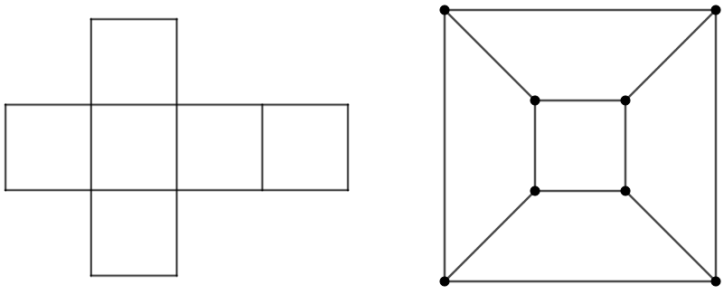


Figure 9: A cube skeleton[2] with the corresponding 2-dimensional representations: net and planar graph

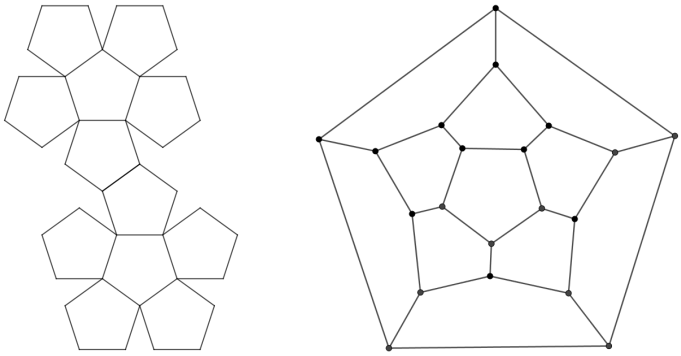


Figure 10: A dodecahedron skeleton[3] with the corresponding 2-dimensional representations: net and planar graph

Students can be asked to compare and discuss the two 2-dimensional representations (net and planar graph): Which of the 2-dimensional representations provide better visualization of the number of edges at each vertex? Of the total number of vertices? Of the total number of edges? Of the total number of faces? Which one gives better visualization of the 3-dimensional object they represent and why? Which one (in your opinion) will help you better in solving the puzzle and why? Etc. These discussions allow students to think critically: how to solve a 3-dimensional complex task and make it simpler (by solving a much more manageable corresponding 2-dimensional task).

Hence, the follow up question to brainstorm with students is: How the planar graph of the dodecahedron can help one to solve the puzzle – to make the 3-colored origami dodecahedron? This leads to formulating the corresponding 2-dimensional puzzle: how to color the edges of the planar graph of the dodecahedron with three colors such that no two graph edges coming out of a graph vertex are colored with same color?

To address this task, let's look at the following puzzle called *Around the World* invented by the Irish mathematician Sir William Rowan Hamilton:

> In 1857, Hamilton introduced a game consisting of a solid regular dodecahedron made of wood, twenty pegs (one inserted at each corner of the dodecahedron), and a supply of strings. Each corner was marked with an important city of the time. The aim of the game was to find a route along the edges of the dodecahedron that passes through each city exactly once and that ended at the city where the route began. In order for the player to recall which cities in a route had already been visited, the string was used to connect the appropriate pegs in the appropriate order. There is no indication that the game was ever successful. (Chartrand and Lesniak, 1996, p. 103).

The corresponding game on the planar graph of the dodecahedron asks: Choose your favorite vertex on the planar graph. Can you start at that vertex, visit all other vertices by traveling along the edges, and come back to the favorite vertex, without visiting a vertex twice (except the favorite one)? Note that the requirement is to visit every vertex, not every edge. The solution to this 2-dimensional *Around the World* game is a circuit that is called a *Hamiltonian circuit*. The game is fun and requires patience, persistence, and sometimes several tries to get to the solution, so some students might be frustrated with the struggle. However, the reward after solving it is gratifying. Also, realizing that there is no unique Hamiltonian circuit is surprising and fascinating to them.

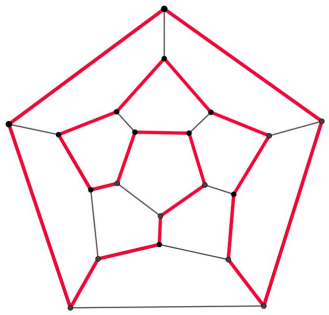
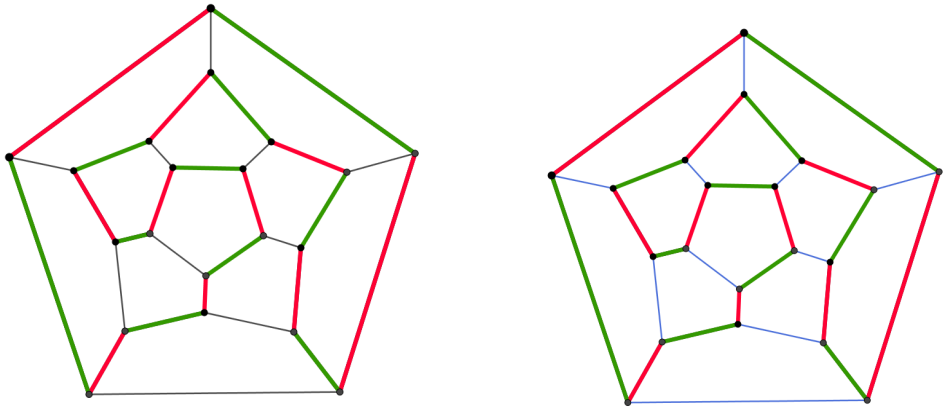


Figure 11: A Hamiltonian circuit

How can this Hamiltonian circuit be used to address and solve our original 2-dimensional puzzle – coloring the edges with 3 colors? Can the edges on this circuit be colored with two colors such that no two edges on the circuit that meet at a vertex have the same color? If so, why can that be accomplished?

The idea is that every Hamiltonian circuit students find will contain 20 edges that can be colored with two colors by alternating the colors (Figure 12a – the edges colored in red and green). What happens with the uncolored edges? Students should be able to notice that coloring the uncolored edges with the third color gives a solution to our 2-dimensional puzzle (Figure 12b – those are the edges colored blue).

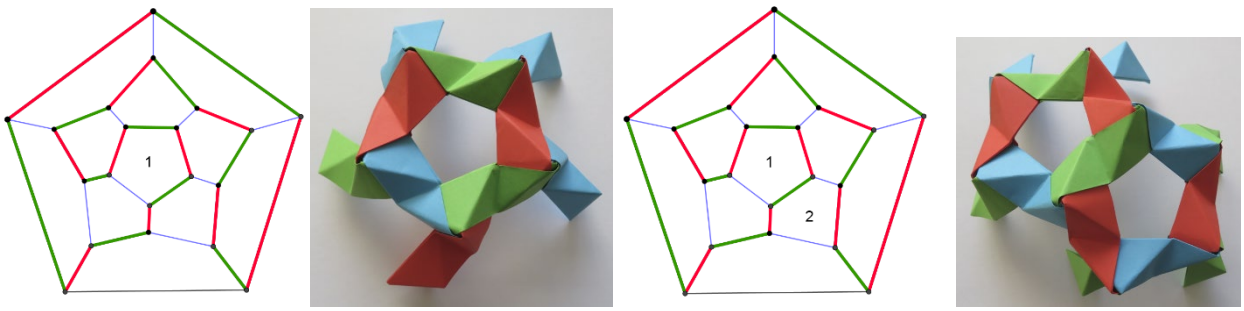


a) b)

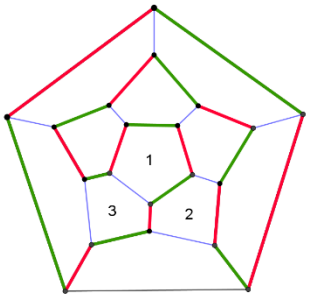
Figure 12: 3-coloring of a planar graph of the dodecahedron

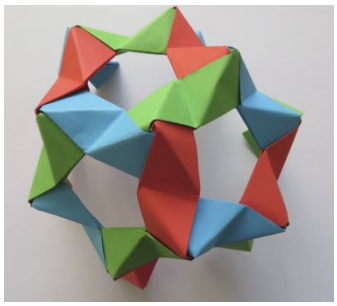
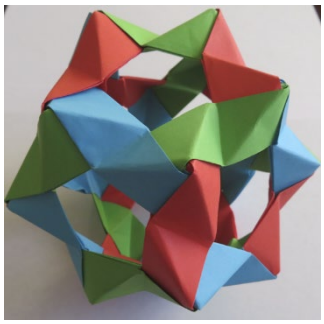
How can one use the solution of the 3-colored planar graph to construct the 3-colored origami dodecahedron?

Using 3-dimensional polyhedra to draw their corresponding 2-dimensional planar graphs and reverse – using these planar graphs to visualize the 3-dimensional solids they represent – is a great way to nurture not just students' visualization but also their imagination. Students should be able to see how each edge of the planar graph corresponds to the dodecahedron edge and start building the origami dodecahedron. Illustrations below show how to proceed building the dodecahedron face by face.



a) Folded Face 1 b) Folded faces 1 and 2



c) Folded faces 1, 2, and 3 d) Finished model

Figure 13: Folding 3-colored origami dodecahedron (face by face) using the 3-colored planar graph

Students might be given task to discuss and observe at which stage of folding the 3-colored dodecahedron they do not need the planar graph anymore (they have just one choice based on the already assembled PHiZZ units). This is a great way for them to play and maybe guess at first and then confirm their findings.

As a challenge assignment, students can be asked to build a 3-colored soccer ball (the left most origami model on Figure 7).

While playing with folding this origami model, students will need a lot of patience and perseverance to complete this task. Some might be struggling along the way. But their curiosity will draw their playful side to finish their final creation. The final object will fill them with gratitude and enchantment, and the realization that math helped them in achieving this task – in folding this beautiful model – is priceless.

Conclusion

The ideas presented in this article can be found elaborated in more details in Hull (2006). As pointed out, I have been using them to provide opportunities for students to wonder, explore, play, and discover through mathematics and origami. I challenge the readers to explore these ideas deeper and explore more of the playful side of mathematics for themselves and for the students they teach (if any).

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The Call to Modernize Mathematics: Equipping Students and Educators with the Skills to Navigate a Data-driven Landscape

By Sydney Tesch and Lindsey Henderson

After spending some time with the Utah State Board of Education's Secondary Mathematics Specialist, Lindsey Henderson, it is easy to become intrigued by data science. With a passion for the topic and its ability to enrich the lives of learners, Lindsey understands the importance of teaching the powerful topic.

Data science is a field of study and/or an area of expertise. Those employed in the field are typically artificial intelligence (AI), machine learning, or programming professionals, data architects and analysts, and data scientists. In its purest form, data science is being inquisitive about data and drawing conclusions to make decisions. While relating, applying, and teaching data science to the K-12 world may seem a daunting task, Lindsey believes that all teachers are capable of encouraging their students to interact with data sets and use critical thinking skills to make decisions about the data.

According to Lindsey, students need data science more than ever. In the past, mathematic super skills were used to compute and determine correct answers to equations. With the evolution of society, it is imperative that mathematics evolve as well. Students can now use calculators, the internet or simply ask "Alexa" for help with square roots. What we need are students who can understand what a square root means and why it matters. The influence of the news and media has a profound impact on belief and opinion which some consider to be fact. The practice of data science in secondary grades allows students to investigate a data set and glean information. It prepares them to critically analyze and understand if said information is presenting the data in a biased way. These kinds of data-driven learning experiences allow students to develop what the Utah State Board of Education's Portrait of Graduate identifies as ideal characteristics, "Digital Literacy, Communication, Critical Thinking and Problem Solving, Creativity and Innovation, and Collaboration and Teamwork" (Utah State Board of Education [USBE], 2019).

Lindsey notes that this hope of the focus on data science does not come without its limitations and barriers. She says, "The holder of the math culture esteems Calculus as the penultimate secondary mathematics course and this can be seen in teacher preparation programs. In the state of Utah, teachers are required to take 4 Calculus classes, but only 2 Statistics classes and 1 Linear Algebra class." She continues, "Secondary mathematics educators seem to feel comfortable teaching Calculus OR Statistics, rarely do you get a crossover. We need to change

these systems so that teachers are comfortable and prepared to teach all maths, including Calculus. I'd love to buoy teacher's confidence in building statistical reasoning by giving them more exposure to data science."

So, what could data science look like in your classroom? It is being vulnerable and giving up a portion of control for students' wonders and questions about data sets. It is being willing to not know the answers and willing to discover solutions with students on their investigative journeys. It is providing engaging opportunities for students to explore mathematics and how it shapes the world around them. Many examples of data science being used in the classroom can be found in the GAISE II Pre-K-12 Report. One of these is focused on students choosing types of music for a school dance. The report explains the process in these steps:

1. Formulate statistical investigative questions: *What type of music do the children in our grade like?*
2. Collect/consider data: *Have students organize and take a survey with different types of music.*
3. Analyze the data: *Students use different charts and graphs to visualize and understand the data.*
4. Interpret results: *Students are prepared to make decisions based on the information they have collected and organized.*

This example provides students with an opportunity to engage in a real-life problem solving activity. They can work individually and/or collaboratively to understand the many ways to understand the data set and find a solution. As mentioned in his book, "Mathematics for Human Flourishing," Francis Su said, "Too often we think of math like a pole in the ground: there's only one way for a vine to grow up the pole. In reality, math is like a trellis: as a vine, you can find your way up at multiple places where the trellis meets the ground, and you can grow in multiple directions along the trellis" (Su, 2021). With this data science problem and others, students can come to understand this idea and develop their mathematical identities while building mathematical critical thinking skills--something that industry keeps telling us is in high demand!

Data science is also an incredible tool for ensuring equitable practices in schools. In a professional learning community, educators have access to a lot of data in the form of assessments, assignments, and more. As these communities collect data on student learning, they can make informed decisions on what their students need. Richard Dufour, educational researcher on K-12 professional learning communities poses four questions that he claims are "crucial to drive the work of the community."

1. What do we want all students to know and be able to do?
2. How will we know if they learn it?
3. How will we respond when some students do not learn?

4. How will we extend the learning for students who are already proficient?

(DuFour, 2006)

With the use of the data, these questions can be deeply considered and answered by any mathematics education professionals engaging in a professional learning community (PLC). By engaging in these data-driven conversations, teachers can ensure that students are more likely to receive an equitable education and develop a healthier relationship with learning.

There are several opportunities available for Utah Educators that will allow participants to distinguish themselves as a leader in Utah's Data Science landscape:

- Utah State Board of Education's Introduction to Data Science Course
- Utah State Board of Education's Data Science Microcredential Stack
- USBE's in Partnership with Stanford University: 21st Century Teaching and Learning Data Science Course

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Syd Tesch (sydney.tesch@schools.utah.gov) is employed at the Utah State Board of Education where she assists in the Student Support Department. With a life-long learner mindset, Syd has searched for opportunities in education both for herself and others. She graduated from Weber State University with a degree in Civic Advocacy and is committed to ensuring access and equity for all students. Syd has a passion for research and statistics and believes that understanding data can bring about positive change. She enjoys reading, finding new music, and spending time with her math obsessed husband and adorable cat.

Lindsey Henderson (lindsey.henderson@schools.utah.gov) currently serves Utah students as the Secondary Mathematics Specialist for the Utah State Board of Education. She is passionate about providing rich learning opportunities for all Utah mathematicians where learners are engaged in building and maintaining strong mathematical identities (a process she refers to as "mathing"). She began her education career teaching all of the secondary maths for Salt Lake City School District (and some science too!). After her time in the classroom, she joined the Salt Lake City educational technology startup ecosystem where she built strong relationships to industry and honed her vision for relevant and meaningful mathematics for all. She returned to public education as the K-12 Mathematics Specialist for Davis School District where her ideas about humanizing mathematics for students were cultivated, supported and allowed to flourish. Lindsey is driven by a strong desire to provide relevant and meaningful mathematical experiences for Utah students and educators, the mathematical teaching, learning, and equity practices, and inquiry-based pedagogy. She lives in the City of Salt with her wife, her children (who are always exasperated to realize that math actually IS everywhere), and her Scottish kilt kittens.

Lesson – Scale Up or down

Don Busenbark – Ed.S. (Utah State University)

Jaden Libberton – (Utah State University)

Utah Core Mathematics Standard: Secondary Mathematics II

Standard G.SRT.1 *Verify experimentally the properties of dilations given by a center and a scale factor*

Standard G.SRT.2 *Given two figures, use the definition of similarity in terms of similar transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.*

Project

Students will be required to make their own model scaling an object of their choice. Students will be given the rubric (see attached rubric) and an explanation of the project

Set up

Select a movie such as "The Lord of the Rings" or "Honey, I Shrunk the Kids" and show the segment on the making of the movie that discusses the models and scale factor. This will help students understand the value of scale factor in making models for movies that are realistic. (I use "The Lord of the Rings – The Two Towers – Bigatures" section of the making of the movie)

Timeline

The project should take the students a couple of weeks for them to complete the task. The task can be completed during class time or out of class. (I usually assign the project after we have started the unit on dilations and similar shapes and the students work on it after class at home). Teachers will need to decide on how much time and whether to assign during class time or out of class time.

Presentation

Students should present their projects in class to their peers. This should take a day (or possibly two days) for students to measure projects and check the accuracy of the scale factors. This reinforces the idea of scale factor and helps the teacher with grading the project. This is also the day for students to vote for the best design and the chance for the top 3 to be awarded a small prize. In addition to the potential extra credit, this competition will encourage more students to be creative with their project.

Scale Project Grading Rubric

Students will construct a scale model. The students must create a 3-dimensional model that is scaled down or up from the original. Examples of objects that students can use to create a

scale model: ipads, cellphones, pencils, automobiles, homes, rooms (such as their bedroom, living room, etc), a building, city, a bridge, etc. Students will be required to include a picture (digital or printed copy) of the structure along with the model. Students will need to include a legend showing the scale of the model. Students also need to include the actual measurements of the object being created as a scaled model. (*Real models, such as car models, will not be accepted.*)

Students may create their own model from cardboard, paper, legos, cooking items or other materials they have available at home. If students need materials for their model, they should visit with the teacher to find out what materials may be available. Students will be graded on completing the assignment on time, following directions, including a legend, measurements, a picture of the original object, and the accuracy of measurements. Students may earn extra credit for the overall appearance of the model and the top 3 projects will be awarded a small prize.

Each project will be measured by at least 3 other students to check for accuracy and the students will award the grade for the measurement portion of the project.

Students must include this rubric sheet with their project.

Projects will be due by _____

The scale model must be accurate. For example, if you do your bedroom and your bed is 6 feet long and the scale is 1-foot equals 1 inch then the bed should be 6 inches long.

Grading Rubric:

Project is turned in on time/early	Late -20	On time/Early 10	
Scale Accurate in project	No Accurate measurements 0 25% of measurements accurate 10 50% of measurements accurate 20 75% of measurements accurate 30 100% of measurements accurate 40		
Picture of modeled object is included	0	10	
Dimensions of original object included	0	10	
Legend included	0	10	
Model appearance	0	10	20 (Extra Credit)
In-class presentation	0	10	

Total points _____

Student measurement grading sheet

Student Name (person who did project) _____

Accurate measurements (Circle the one that best represents how well the student did with their scale factor)

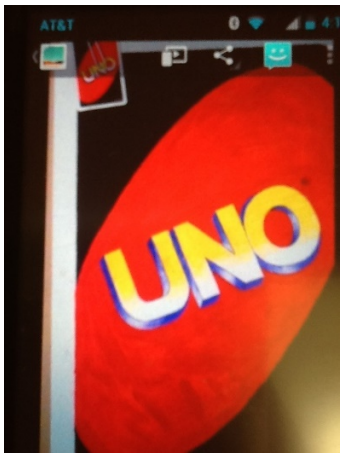
0% accurate 25% accurate 50% accurate 75% accurate 100% accurate

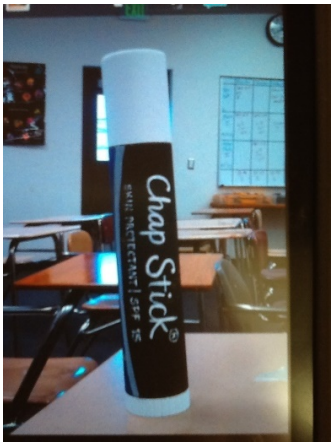
Overall impressions of project

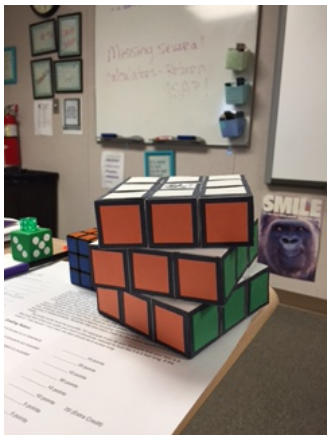
_____

Your name (Student who graded project)_____

Examples of Student projects






SMILE

Professional Learning for Elementary Mathematics Specialists

Shannon Olson (Alpine School District)
(Shannon was serving as the Elementary Mathematics Specialist for the Utah State Board of Education when the article was written.)

Abstract

This action research study examined the impact professional learning had on the confidence and effectiveness of Elementary Mathematics Specialists. The topic was selected after a review of literature that discussed the transition from classroom teacher to Elementary Mathematics Specialist and the need for these specialists to receive professional learning specific to their roles. The research question was: How does providing professional learning opportunities to Elementary Mathematics Specialists impact their confidence and effectiveness in their roles in schools and districts? Forty aspiring and current Elementary Mathematics Specialists participated in professional learning and were asked to complete a survey on how the opportunity impacted their confidence and effectiveness across several learning goals. The mixed-methods approach analyzed responses related to the confidence and effectiveness gained from the professional learning experience. The study suggests that providing Elementary Mathematics Specialists with professional learning does increase their confidence and effectiveness in their roles.

Topic

Elementary Mathematics Specialists are educators that specialize in content, pedagogy, and leadership at the elementary level. Some Elementary Mathematics Specialists serve as coaches, some serve as interventionists, and others serve as teachers solely responsible for mathematics instruction at the elementary level.

A recent publication from the National Council of Teachers of Mathematics calls for the role of Elementary Mathematics Specialists in supporting student equity and achievement. “Catalyzing change in early childhood and elementary mathematics raises issues for critical examination by all stakeholders and calls for systemic approaches that ensure access, equity, and excellence for each and every child. Stakeholders create tremendous power by talking with one another and working together. ... Stakeholders in early childhood and elementary mathematics include classroom teachers, instructional coaches, specialists, and leaders…” (Huinker, 2020).

The role of an Elementary Mathematics Specialist may have a positive impact on student outcomes. Research suggests there is limited, but existing evidence that having specialists in elementary mathematics may lead to student gains in understanding and mathematics proficiency. A study involving 32 teachers that solely teach mathematics in highly-impacted schools in a large urban district while receiving intensive training in mathematics content shows positive outcomes. Student achievement increased including more students responding to

performance-based assessment items rather than not answering questions as many did on a given pretest (Nickerson, 2010).

There is a transition from being a classroom teacher to an Elementary Mathematics Specialist. Being a specialist requires a different skill set than being a teacher and it takes time and practice for an educator to acquire the role of being an effective specialist.

There are positive impacts on Elementary Mathematics Specialists when they receive professional learning. Professional learning may include an emphasis on mathematics content and pedagogy as well as on leadership and coaching skills. Specialists report having more confidence when provided with professional learning. In MaGatha's review of research and practice on Elementary Mathematics Specialists, she shares that several national mathematics education organizations have developed publications in support of the role over several decades. The research clarifies various job capacities an Elementary Mathematics Specialist may take on and describes standards that support specialists in performing those capacities. She addresses a need for professional learning for specialists as there is a transition from classroom teacher to specialist (McGatha, M. B., 2017).

Districts, universities, and organizations advocate for the existence of the role of Elementary Mathematics Specialist as well as for professional learning for those assuming the role. There is a need for sustainable professional learning in correlation with positive impact on student outcomes. In order to see positive outcomes on teachers being coached by a specialist, the coach needs to be full-time rather than part-time. In order for specialists or coaches to gain the skills necessary they must also have sustained and intensive professional learning.

The topic of professional learning for Elementary Mathematics Specialists is relevant to the field of education because it impacts student learning in elementary mathematics. About 50% of students in Utah are proficient in mathematics (Utah State Board of Education, 2021). Elementary Mathematics Specialists and coaches have opportunities to work with teachers in districts and schools. As these specialists are better prepared with their own professional learning, they may be more effective in working with teachers, which may ultimately improve student outcomes in elementary mathematics. This study will ask the participating Elementary Mathematics Specialists to report on how participating in professional learning has impacted their effectiveness at the district and school levels.

Problem Statement

High-quality teachers are often hired to serve in roles as mathematics specialists or coaches, but are often not provided professional learning on assuming their new roles. Consequently, specialists may not stay in their new role very long because they feel ill-prepared. They may also struggle to build trust with the teachers they serve because they do not have the knowledge and skills required for the role.

Problem Background and Causes

Being a specialist requires a different skill set than being a teacher and it takes time and practice for an educator to acquire the role of being an effective specialist. "Once on the job,

EMSs find that it is not obvious what to do or how to do it. Therefore, EMSs require support to be able to hone their skills and be successful in working with students and/or teachers to implement research-based best practices to optimize student learning" (McGatha, 2017). The support that Elementary Mathematics Specialists require and have access to varies widely across the nation.

District and school leaders responsible for Elementary Mathematics Specialists may not realize the need for professional learning related to the new role since most specialists were expert teachers. Leaders may also lack resources to provide professional learning or may not understand the amount of time needed to develop new skills.

"A three-year study found that those responsible for coaching math teachers positively affected student academic progress in grades 3, 4, and 5." The study took time to emerge because of the need for Elementary Mathematics Specialists to transition from classroom teachers to specialists. "The positive effect of elementary mathematics specialists on student achievement did not occur simply by creating a position and naming an individual to serve as a school's specialist. It developed overtime as a knowledgeable elementary mathematics specialist and a school's instructional and administrative staff learned and worked together. … These math specialists were accomplished classroom teachers who enjoyed teaching mathematics to children; but to be a specialist and a coach, they had to draw on other skills and proficiencies. To support that transition as prospective specialists, they completed five mathematics content courses and two leadership/coaching courses" (Campbell & Malkus, 2013). Elementary Mathematics Specialists need training in content, pedagogy, and leadership to effectively serve in the role.

Districts and charter schools in Utah have varying models for the use of Elementary Mathematics Specialists. There are veteran, new, and aspiring specialists who were expert teachers and may not have been trained in coaching or serving in a district-level specialist role. Several specialists throughout the state are in the process of transitioning to new roles and need professional learning in supporting them with the transition.

Research Question

How does providing professional learning opportunities to Elementary Mathematics Specialists impact their confidence and effectiveness in their roles in schools and districts?

Topic and Problem Conclusion

Elementary Mathematics Specialists are critical in supporting student outcomes in mathematics; however, some specialists do not feel as confident and are not as effective as they have the potential to be. Professional learning is needed to support Elementary Mathematics in the transition from teacher to specialist. This study seeks to identify how providing professional learning opportunities to current and aspiring Elementary Mathematics Specialists impacts their confidence and effectiveness in their roles.

Overview of the Literature

Elementary Mathematics Specialists are educators that specialize in content, pedagogy, and leadership at the elementary level. Some Elementary Mathematics Specialists serve as coaches, some serve as interventionists, and others serve as teachers solely responsible for mathematics instruction at the elementary level. The role of an Elementary Mathematics Specialist may have a positive impact on student outcomes. Research suggests there is limited, but existing evidence that having specialists in elementary mathematics may lead to student gains in understanding and mathematics proficiency. There is a transition from being a classroom teacher to an Elementary Mathematics Specialist. Being a specialist requires a different skill set than being a teacher and it takes time and practice for an educator to acquire the role of being an effective specialist. There are positive impacts on Elementary Mathematics Specialists when they receive professional learning. Professional learning may include an emphasis on mathematics content and pedagogy as well as on leadership and coaching skills. Specialists report having more confidence when provided with professional learning.

Districts, universities, and organizations advocate for the existence of the role of Elementary Mathematics Specialist as well as for professional learning for those assuming the role. There is a need for sustainable professional learning in correlation with positive impact on student outcomes. In order to see positive outcomes on teachers being coached by a specialist, the coach needs to be full-time rather than part-time. In order for specialists or coaches to gain the skills necessary they must also have sustained and intensive professional learning.

Elementary Mathematics Specialist Impact on Student Outcomes

A recent publication from the National Council of Teachers of Mathematics calls for the role of Elementary Mathematics Specialists in supporting student equity and achievement. “Catalyzing change in early childhood and elementary mathematics raises issues for critical examination by all stakeholders and calls for systemic approaches that ensure access, equity, and excellence for each and every child. Stakeholders create tremendous power by talking with one another and working together. ... Stakeholders in early childhood and elementary mathematics include classroom teachers, instructional coaches, specialists, and leaders. ...” (Huinker, 2020).

Studies suggest the impact a specialist has on student outcomes is connected to the amount of access the specialist has to working with teachers as well as the activities the specialist engages in. One study suggests positive outcomes on student results when there are full time Elementary Mathematics Specialists in schools, but not part time specialists. In the same study “results revealed statistically significant positive relationships between the time (specialists) spent on the two responsibilities related to supporting teachers (i.e., supporting both individuals and groups of teachers) and student achievement, but there was either no relationship or a negative relationship noted for their work with students” (Harbour & Saclarides, 2020).

A study involving 32 teachers that solely teach mathematics in highly-impacted schools in a large urban district while receiving intensive training in mathematics content shows positive outcomes. Student achievement increased including more students responding to performance-based assessment items rather than not answering questions as many did on a given pretest (Nickerson, 2010).

In describing a university-school district partnership (Rigelman, Crane, Kellie Petrick, & Shrier, 2018) state, “The most productive use of a district mathematics specialist’s time is within schools, working directly with teachers and students.” They explain, “The mathematics specialist and school principal collaborated to sustain and propel this learning culture beyond the participating grade-level teams. They scheduled schoolwide professional learning —for example, on how to implement problem-based lessons and how to deepen mathematical discourse with connections to the district’s instructional framework — both to minimize initiative fatigue and generate broader enthusiasm for the work.” Connections may be drawn that the positive results of the partnership on student outcomes may in part be connected to the role of the mathematics specialist..

Transition from Classroom Teacher to Elementary Mathematics Specialist

“A three-year study found that those responsible for coaching math teachers positively affected student academic progress in grades 3, 4, and 5.” The study took time to emerge because of the need for Elementary Mathematics Specialists to transition from classroom teachers to specialists. “The positive effect of elementary mathematics specialists on student achievement did not occur simply by creating a position and naming an individual to serve as a school’s specialist. It developed overtime as a knowledgeable elementary mathematics specialist and a school’s instructional and administrative staff learned and worked together. … These math specialists were accomplished classroom teachers who enjoyed teaching mathematics to children; but to be a specialist and a coaching, they had to draw on other skills and proficiencies. To support that transition as prospective specialists, they completed five mathematics content courses and two leadership/coaching courses” (Campbell & Malkus, 2013).

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In MaGatha’s review of research and practice on Elementary Mathematics Specialists, she shares that several national mathematics education organizations have developed publications in support of the role over several decades. The research clarifies various job capacities an Elementary Mathematics Specialist may take on and describes standards that support specialists in performing those capacities. She addresses a need for professional learning for specialists as there is a transition from classroom teacher to specialist (McGatha, M. B., 2017).

Elementary Mathematics Specialists and Professional Learning

Professional learning may include an emphasis on mathematics content and pedagogy as well as on leadership and coaching skills. Specialists report having more confidence when provided with professional learning. According to the results of a university-based program for Elementary Mathematics Specialists in Oklahoma, "the experiences and coursework as part of the elementary mathematics specialist program have an impact on teachers' leadership skills, abilities, and activities" (Utley & Reeder, 2016).

In a similar study the University of Missouri provided Elementary Mathematics Specialist certification for school-based teams. "A cross-case comparison revealed multiple trajectories for the groups of fellows. While five of six school-based teams named goals for their own classroom instruction in the first iteration of their plan, not one team applied a schoolwide lens. However, on the fourth iteration, all six teams were seen to employ a schoolwide lens. Interestingly, we found that only one school had maintained the same focus on "eliciting student thinking/ providing student feedback" from the first to fourth version of their action plan, and that particular team additionally named that focus at a schoolwide level on the fourth iteration" (Dames & Webel, 2020). This example indicates that providing professional learning may help Elementary Mathematics Specialists shift their perspective from the individual classroom level to a school-wide level and may also help specialists become more specific in their mathematics goals.

In addition to Elementary Mathematics Specialist certifications and university-based programs, specialists may benefit from other forms of professional learning. One study "facilitated professional development (PD) in which coaches, principals, and teachers studied mathematics teaching and learning together. Our initial focus on teacher decision-making was inadequate in meeting instructional leaders' learning needs. We adapted the PD to focus instructional leaders' attention on the work of learning teaching. Analysis of leaders' discourse revealed shifts from noticing teacher characteristics to noticing dilemmas and decision-making within teaching and coaching. Findings suggest new roles for teacher educators and new forms of PD for instructional leaders" (Carlson, Heaton & Williams, 2017). There is evidence that job-embedded professional learning for coaches and specialists may transform their practice.

Conclusion

Establishing the role of Elementary Mathematics Specialists in schools may have a positive impact on student achievement. It is more likely to achieve positive outcomes when the specialist is full-time in a school, collaborates with other stakeholders, is provided time and supports to transition from the role of a teacher to a specialist, and is provided with professional learning opportunities. Professional learning may include an Elementary Mathematics Specialist certification or university-based program based in content, pedagogy, and leadership. In addition to certification, Elementary Mathematics Specialists benefit from professional learning in which they may apply strategies in the schools they serve.

Research Design

The purpose of this action research study is to identify how providing professional learning opportunities to current and aspiring Elementary Mathematics Specialists impacts their

confidence and effectiveness in their roles. This study examines the impact of professional learning on practicing and aspiring Elementary Mathematics Specialists who participated in professional learning provided by the Utah State Board of Education in the 2020-2021 school year. Participants were provided with professional learning including synchronous Zoom meetings as well asynchronous readings and discussions in Canvas. Topics for the professional learning included defining the role of an Elementary Mathematics Specialist; building relationships with teachers, administrators, and other stakeholders; deepening knowledge of content and standards including progressions across grade levels; working with individual teachers, grade-level teams, and school faculties; and equity in mathematics education.

This is a mixed methods action research design including collection and analysis of quantitative and qualitative data. After having engaged in the professional learning, participants were asked to complete a brief survey consisting of quantitative measures with Likert scales and qualitative open-ended questions. The study may help the researcher and other stakeholders acquire insights about the impacts of professional learning offered to current and aspiring Elementary Mathematics Specialists.

Research Question

How does providing professional learning opportunities to Elementary Mathematics Specialists impact their confidence and effectiveness in their roles in schools and districts?

Participants

Participants included current and aspiring Elementary Mathematics Specialists who work for districts and charter schools in the state of Utah. There were a total of 40 participants from multiple sites across the state. Of the 40 participants 32 were currently employed by school districts, while eight worked for charter systems. Participants were from various regions across the state with the most northern participant being from Box Elder County and the most southern from Washington County. Some participants were from rural areas in the state, while the majority were located in the Wasatch front from a variety of districts and charters. Some participants served a single charter school and others served districts with up to 60 elementary schools.

Three participants were male and 37 were female. Participants ranged from having seven years up to over 20 years of experience in education. Twenty-seven participants were currently serving in a specialist role at the school or district level. Thirteen participants were currently classroom teachers aspiring to be Elementary Mathematics Specialists. While some participants had not yet served in a specialist role, others were in their first year, and some had served in the role for up to ten years. Twenty-nine participants had earned Utah's 18-credit Elementary Mathematics Endorsement that focuses on content and pedagogy, but not leadership. One participant had taken some of the endorsement courses. Ten participants had not taken any of the endorsement courses. Participants varied in regional area, the roles they served, and the amount of experience they had in their roles.

Data Collection Instruments and Methods

A Qualtrics survey was developed by the researcher with quantitative and qualitative items. The survey began by collecting demographic information such as if the participant is employed by a district or charter, the current role the participant serves in, and the number of years in the role. Next, the survey asked questions with a Likert scale indicating the degree to which they gained confidence or effectiveness in various skills. Finally, the survey concluded with open ended questions asking participants to describe the most impactful components of professional learning they have experienced and specific ways in which professional learning has impacted their work.

The survey is appropriate for the research question as it allowed the participants to indicate how participating in the professional learning for Elementary Mathematics Specialists has impacted their confidence and effectiveness in their roles in schools and districts. The survey asked participants to share what characteristics of the professional learning have been most impactful.

Data Security and Confidentiality

The data gathered from this research is private and confidential. Participants were not asked to provide their name when they completed the survey response. They acknowledged a consent form at the time of completing the electronic survey. Data is reported in the aggregate.

Conclusion

This mixed methods action research study sought to address an issue of the need to provide professional learning to aspiring and current Elementary Mathematics Specialists. Participants from across the state of Utah were invited to attend professional learning specific to the role of Elementary Mathematics Specialist and aligned to topics relevant to the role. Participants were asked to complete a survey with quantitative and qualitative items measuring the impact the professional learning had on their confidence and effectiveness in their roles in schools and districts. The data was analyzed by the frequency and percentages of rankings to describe degree to which participants increased in confidence and effectiveness. Frequency of responses in open-ended questions were used to describe themes related to the way professional learning has impacted Elementary Mathematics Specialists.

Action Research Overview

Elementary Mathematics Specialists are educators that specialize in content, pedagogy, and leadership at the elementary level. There is a transition from being a classroom teacher to an Elementary Mathematics Specialist. Being a specialist requires a different skill set than being a teacher and it takes time and practice for an educator to acquire the role of being an effective specialist. There are positive impacts on Elementary Mathematics Specialists when they receive professional learning. Professional learning may include an emphasis on mathematics content and pedagogy as well as on leadership and coaching skills. Specialists report having more confidence when provided with professional learning. In order for specialists or coaches to gain the skills necessary they must also have sustained and intensive professional learning.

This study answered the research question, "How does providing professional learning opportunities to Elementary Mathematics Specialists impact their confidence and effectiveness in their roles in schools and districts?"

This mixed methods action research study sought to address an issue of the need to provide professional learning to aspiring and current Elementary Mathematics Specialists. Participants from across the state of Utah were invited to attend professional learning specific to the role of Elementary Mathematics Specialist and aligned to topics relevant to the role. Participants were asked to complete a survey with quantitative and qualitative items measuring the impact the professional learning had on their confidence and effectiveness in their roles in schools and districts. Thirty of the 40 participants completed the survey.

Findings

Quantitative and qualitative data were collected in a survey with items asking about ways in which the professional learning experience for Elementary Mathematics Specialists impacted their confidence and effectiveness in their roles. The figures and narrative below describe the data collected.

Figure 1: Confidence Table

Participation in the professional learning for Elementary Mathematics Specialists increased my CONFIDENCE in …	Strongly agree	Somewhat agree	Neither agree nor disagree	Somewhat disagree	Strongly disagree
building relationships with teachers, administrators, and other stakeholders	15, 50%	13, 43%	1, 3%	1, 3%	0, 0%
knowledge of content and standards including progressions across grade levels	19, 63%	8, 27%	2, 7%	1, 3%	0, 0%
facilitating coaching cycles with individual teachers in elementary mathematics	17, 57%	9, 30%	3, 10%	1, 3%	0, 0%
working with grade-level teams and school faculties in elementary mathematics	16, 53%	11, 37%	2, 7%	1, 3%	0, 0%
advocating for and supporting equity in mathematics education	18, 60%	8, 27%	3, 10%	1, 3%	0, 0%

Values in the table refer to the number of responses and percent of responses.

According to Fig.1. most participants strongly agreed or somewhat agreed that their confidence increased in each of the constructs considered, while a few participants were neutral or somewhat disagreed about their confidence increasing. Increasing confidence in knowledge of content and standards including progressions across grade levels had the highest response for "strongly agree" with 19 participants or 63% of participants strongly agreeing. Confidence in advocating for and supporting equity in mathematics education was the next highest with 18 participants or 60% of participants strongly agreeing. Confidence in building relationships with teachers, administrators, and other stakeholders had the least amount of participants, 15 participants and 50% of participants, strongly agree. In each construct at least 50% of participants strongly agreed that their confidence had increased. In each construct at least 87% of participants strongly agreed or somewhat agreed that their confidence had increased. One to three participants were neutral on various constructs and one participant somewhat disagreed with increasing confidence in all of the constructs. No participants strongly disagreed with any constructs.

Figure 2: Effectiveness Table

Participation in the professional learning for Elementary Mathematics Specialists increased my EFFECTIVENESS in …	Strongly agree	Somewhat agree	Neither agree nor disagree	Somewhat disagree	Strongly disagree
building relationships with teachers, administrators, and other stakeholders	15, 50%	11, 37%	3, 10%	1, 3%	0, 0%
knowledge of content and standards including progressions across grade levels	16, 53%	10, 33%	3, 10%	1, 3%	0, 0%
facilitating coaching cycles with individual teachers in elementary mathematics	17, 57%	7, 23%	4, 13%	2, 7%	0, 0%
working with grade-level teams and school faculties in elementary mathematics	13, 43%	11, 37%	5, 17%	1, 3%	0, 0%
advocating for and supporting equity in mathematics education	15, 50%	9, 30%	5, 17%	1, 3%	0, 0%

Values in the table refer to the number of responses and percent of responses.

According to Fig. 2, most participants strongly agreed or somewhat agreed that their effectiveness increased in each of the constructs considered, while a few participants were neutral or somewhat disagreed about their effectiveness increasing. Increasing effectiveness in facilitating coaching cycles with individual teachers in elementary mathematics had the highest response for "strongly agree" with 17 participants or 57% of participants strongly agreeing. Effectiveness in knowledge of content and standards including progressions across grade levels was the next highest with 16 participants or 53% of participants strongly agreeing. Effectiveness in working with grade-level teams and school faculties in elementary mathematics had the least amount of participants, 13 participants and 43% of participants, strongly agree. In each construct at least 43% of participants strongly agreed that their effectiveness had increased. In each construct at least 80% of participants strongly agreed or somewhat agreed that their effectiveness had increased. Three to five participants were neutral on various constructs and one to two participants somewhat disagreed with increasing effectiveness in the constructs. No participants strongly disagreed with any constructs.

Figure 3: Confidence Compared to Effectiveness

Participation in the professional learning for Elementary Mathematics Specialists increased my	CONFIDENCE in …	EFFECTIVENESS in …	Difference
building relationships with teachers, administrators, and other stakeholders	4.4	4.33	0.07
knowledge of content and standards including progressions across grade levels	4.5	4.37	0.13
facilitating coaching cycles with individual teachers in elementary mathematics	4.4	4.3	0.1
working with grade-level teams and school faculties in elementary mathematics	4.4	4.2	0.2
advocating for and supporting equity in mathematics education	4.43	4.27	0.16

Average	4.43	4.29	0.14

Scaled scores were calculated by averaging the responses with the following numerical values: Strongly agree (5), Somewhat agree (4), Neither agree nor disagree (3), Somewhat disagree (2), Strongly disagree (1)

According to Fig. 3, the average response to each construct for confidence and effectiveness fell between 4 and 5, meaning most participants strongly agreed or somewhat agreed with each construct. On average participants had the most confidence in knowledge of content and standards including progressions across grade levels. The had equal averages for the least confidence in building relationships with teachers, administrators, and other stakeholders, facilitating coaching cycles with individual teachers in elementary mathematics, and working with grade-level teams and school faculties in elementary mathematics. Participants had the most effectiveness in knowledge of content and standards including progressions across grade levels. They had the least effectiveness in working with grade-level teams and school faculties in elementary mathematics. Overall participants indicated they increased in more confidence than effectiveness with the average rating for confidence being 4.43 and the average rating for effectiveness being 4.29 with a difference of 0.14. The greatest variability between confidence and effectiveness was with working with grade-level teams and school faculties in elementary mathematics. This construct had an average rating of 4.4 in confidence and 4.2 in effectiveness with a difference of 0.2. The least variability between confidence and effectiveness was with building relationships with teachers, administrators, and other stakeholders 4.4. This construct had an average rating of 4.4 in confidence and 4.33 in effectiveness with a difference of 0.07.

Figure 4: Comments Table

Question: In what additional ways did participation in the professional learning for Elementary Mathematics Specialists impact your confidence and effectiveness in your role as an Elementary Mathematics Specialist at the school or district level?

This course connected me with other Math specialists across the state. This has been very beneficial.
It helped me to understand a little better what my role as a math specialist was, but I don't feel that I gained good solid skills that helped me be more effective in my role.
Through this program I was able to better understand the role of a specialist and will better be able to advocate for that position in the future.
I feel that the role has been better defined for me, and I have a better idea of what I might be experiencing in schools if I were to go into a coaching position.
I have resources available to implement next year.

Though there are many things I don't know and still need to learn, this professional learning opportunity has helped me know where to look for information when I want to learn more. The resources shared will be of great value to me.
There were many things that validated what I am currently doing as a Math Specialist. It gave me things to consider to increase my effectiveness in my role however, and areas I can work on.
Being a math specialist has felt like a very unique and lonely job. The networking with other professionals with similar jobs, beliefs and goals has been very powerful. I feel like I now have people to ask and work with.
It helped increase my confidence in being able to communicate with my principal about the role of a mathematics specialist. It also gave me ideas for approaching and successfully working with all types of teachers and students.
Participation gave me a broader perspective of the role of math specialist and the ways that the role is used in different districts and charters. I appreciated the opportunity to collaborate with specialists across the state.
I appreciate having resources to return to when I need to refresh my knowledge on a particular challenge. I also valued the conversations with others and realized that the challenges that I face are common for many!
I'm not an EMS yet, but my participation in this EMS professional learning opened up my eyes to matters and resources I would never have considered or reflected upon were it not for this opportunity.
Mostly, it helps me to work with the math teachers at my school in a way that helps me collaborate with them to support special education students in their classrooms.
I understand better the many roles that a specialist can have. I also understand a major component of this is building relationships of trust with all stakeholders.
I was able to work with others and discuss the vertical alignment of standards and meet with others who have experience in different grade levels.
This Professional learning helps me clarify the role that Mathematics Specialist should be. I also learned that what skills a specialist needs in the aspect of the curriculum and relationship with the school personnel.
I just felt much more prepared with research behind strategies and reasoning for how mathematics instruction should be approached. So whenever I was involved in a discussion, I felt able to speak to the effectiveness of the research based strategies and how we need to move forward as educators.
I was shown so many new resources that I didn't even know about and also made connections with other people that I didn't know before but all have aspirations to be math specialists.

I like have current information and being able to network with other specialists. I have reached out to others in their role to ask questions, which occurred because of my participation in the trainings and relationships we have built during interactive sessions.
I was able to interact with teachers to discuss mathematics in ways that I hadn't previously
Confidence in finding standards, skills, and knowledge needed in each grade so that I can help teachers and teams in each grade level.

Participants shared several additional ways in which the professional learning increased their confidence and effectiveness. Most participants shared areas in which their capacity was built without distinguishing confidence or effectiveness, while a few mentioned confidence. Nine participants spoke to their capacity being built in understanding the role of an Elementary Mathematic Specialist. Seven participants mentioned the opportunities for collaboration and networking. Five participants reported having the ability to identify and use more resources. Four participants mentioned having gained better ways to communicate and build relationships with stakeholders. Three participants mentioned gaining a better understanding of mathematics instruction while two mentioned an increased ability to work the mathematics standards.

Answers to the Research Questions

Overview

The role of an Elementary Mathematics Specialist requires a different skillset than that of a classroom teacher. When a teacher transitions from being a teacher to becoming a specialist, they may be supported by professional learning specific to their role. Providing Elementary Mathematics Specialists with professional learning may increase their confidence and effectiveness in their roles. In this study specialists were provided professional learning including synchronous Zoom meetings as well asynchronous readings and discussions in Canvas. Topics for the professional learning included defining the role of an Elementary Mathematics Specialist; building relationships with teachers, administrators, and other stakeholders; deepening knowledge of content and standards including progressions across grade levels; working with individual teachers, grade-level teams, and school faculties; and equity in mathematics education. Participants had a greater increase in confidence than they did in effectiveness. They gained the most confidence and effectiveness in knowledge of content and standards including progressions across grade levels.

Problem Solutions

The research problem is that high-quality teachers are often hired to serve in roles as mathematics specialists or coaches, but are often not provided professional learning on assuming their new roles. Consequently, specialists may not stay in their new role very long because they feel ill-prepared. They may also struggle to build trust with the teachers they serve because they do not have the knowledge and skills required for the role. The action research study provide a solution to this problem, at least in part. Providing specialists with professional learning may

increase their confidence and effectiveness in the knowledge and skills required for the position. As elementary mathematics teachers feel more confident and become more effective in their roles, they will likely have better success in working with teachers, may stay in their roles longer having the potential to impact student outcomes.

Limitations

Limitations of the study include the variance in background of the Elementary Mathematics Specialists who participated in the study. There were aspiring specialists, specialists within their first year or two in their roles, and specialists who had served in their roles for several years. Aspiring specialists were not necessarily able to be effective with some of the skills because they had limited opportunities to utilize them. Veteran specialists may not have increased as much in confidence and effectiveness because they may have already had a substantial amount of confidence and effectiveness.

Another limitation is that the specialist roles and responsibilities varied widely across districts and charters. Some specialists were primarily responsible for student interventions, others were coaches at one or a few schools, and yet others were specialists at the district level with many schools. While many participants shared that they loved collaborating across types of roles, some indicated they would have preferred to collaborate with role-alike colleagues rather than such a diverse group.

Forty participants engaged in the professional learning and only thirty participants completed the survey. The responses were anonymous so it is unclear how who responded and did not respond may have skewed the data.

Implications

The implications of this study include the support for providing professional learning for Elementary Mathematics Specialists. Schools and districts often hire Elementary Mathematics Specialists without providing them with professional learning specific to their role. District level leaders may provide professional learning for specialists. Principals may encourage specialists to attend professional learning offered by the district, state, or professional organizations. The state may provide professional learning for Elementary Mathematics Specialists across districts and charters in the state. As Elementary Mathematics Specialists engage in professional learning they may become more confident and effective in their roles having better relationships and outcomes in their schools and districts.

Further Investigation

Further investigation could involve studying the format and delivery of professional learning for Elementary Mathematics Specialists. A researcher could study outcomes for synchronous, asynchronous, virtual, and face-to-face opportunities. The frequency, duration, and ongoing nature of the professional learning opportunities could be investigated. Differentiation for various types of Elementary Mathematics Specialists could be investigated. A researcher could study what a novice specialist needs compared to a veteran specialist. Studies could also investigate the needs of school-based coaches compared to district-level specialists.

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Insights into a Precalculus Exercise

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Abstract: A one-to-one function is a concept known to mathematics teachers. That is why I am not going to state this definition again, trying to demonstrate how a well-chosen example might trigger positive mathematical attitudes. We, as teachers, encounter both types of students' attitudes, positive and negative. I intend to encourage the former and minimize the latter through an example exercise. An exercise based on the one-to-one concept is presented with accompanying comments compatible with the sociocultural theory by Vygotsky and an emotion model, the OCC (Ortony, Clore & Collins, 1988). I aspire to attract the interest of math teachers not only on the corresponding math level but on any level to incorporate in their reality the aforementioned tools.

Teaching Episode

Exercise

i) Examine whether the function f with $f(x) = e^x + x$ is one-to-one

ii) Solve the equation $e^{x^2-x} - x = e^6 + 6 - x^2$.

Usually, the proof of a one-to-one function requires showing that

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

for all x_1, x_2 in the domain of f "(Frangoulopoulos & Kontou,2018, p.97).

But in our case, this seems impossible. A wonderful opportunity to present to my students the value of the mathematical thinking. Value that is directed not only to the mathematical teaching itself but also and primarily to the dialogue that takes place in a math class. I believe it will really be challenging for both teachers and students to unveil what they approve and what they disapprove, how they react when they approve and how they behave themselves when they disapprove. Thus, teachers encourage the mathematical dialogue and urge students to understand what can be deemed valuable, praiseworthy in a stricter context. At the same time the dialogue process as an everyday practice is consolidated in them. Again, here we underpin the importance of language in a sociocultural context the way Vygotsky states it "language the very means by which reflection and elaboration of experience takes place, is a highly personal and at the same time a profoundly social human process" (Vygotsky, 1978, p.126). Language into math classroom really *makes mathematics flourish* since it addresses not only those who have a knack for it but also those who consider mathematics as drudgery. Let us not forget the phrase "we are in this boat together" (Glaze, 2020, p.6).

Let me go back to the exercise. I keep saying to the students there is another way, an alternative, which can be connected to an analogous thinking in everyday life. Mathematics is within life, within the society and I mention here how influenced I am by the sociocultural theory of Vygotsky (Vygotsky,1978). Even the words in the title of his book are enough to highlight the

importance of social interaction, *mind in society*. The time we devote to solve a mathematical problem resembles the time we dedicate to cope with an everyday problem, an everyday dilemma. Given the analogies to the everyday encountered situations, students will be led to appreciate, to approve the advantages of the mathematical thought. Terms like approve and praiseworthiness refer to the second pillar of our emotion OCC model, namely *standards*. Let me mention here that there is no presumed order among the pillars of the OCC, I just keep the order of a paragraph I have included in the presentation of the theoretical model below (Ortony & Clore, 2015, p.310). But standards involve *agents*, not restricted to, but surely referring to persons too. Let me emphasize here that both students and teachers are agents of our environment, of our surroundings, again a component of the sociocultural theory.

I am now ready to proceed to the solution of part a of the exercise following some steps that will lead me to my initial *goal*, the first pillar of the OCC model. A goal that provides both the student and the teacher with the pleasure of achievement. Whatever I have written above is not a monologue, it becomes the essential part of a dialogue with students, where they become active players.

Solution of part i)

$$\begin{gathered} x_1 < x_2 \\ e^{x_1} < e^{x_2} \\ x_1 + e^{x_1} < x_2 + e^{x_2} \\ f(x_1) < f(x_2) \end{gathered}$$

The other path here, the alternative was the monotonicity which implies that f

is one-to-one.

I seem to somehow ignore whether the students I have before me *like* or *dislike* mathematics, the appeal of mathematics to them. The third pillar of the OCC model, *tastes*. We, as teachers, have certain preferences over the teaching material, we tend to like specific parts even of an exercise and sort of dislike others. Students on their sides have their own likes and dislikes on a given example. The interchange of this tastes' attitudes can be really fruitful while solving a problem and can create positive feedback for similar or even more difficult examples. This can lead to a broad discussion among educators of any level whether they can take into account the appeal of their teaching subject, not just restricted to mathematics, to their audience. Sharing with our students the acknowledgment of the like/dislike factor proves to precipitate the whole teaching process.

Solution of part ii)

$$\begin{gathered} e^{x^2-x} - x = e^6 + 6 - x^2 \\ e^{x^2-x} + x^2 - x = e^6 + 6 \\ f(x^2 - x) = f(6) \\ x^2 - x = 6 \\ x^2 - x - 6 = 0 \\ x = -2 \; or \; x = 3 \end{gathered}$$

I pose the students the question how they would have reacted had I examined the part ii of the exercise on its own. A typical, but surely hard to solve equation. My intention is to underpin

the importance of associating concepts, in my case the general concept of a function and specifically, the notion of the one-to-one function. They surely have encountered difficult equations to solve. But here I pose the goal-a pillar of the OCC model- to bring the one-to-one function into the game. I believe they will be pleased, at least a certain percentage of them, to see the applicability of a strong concept, like the one-to-one function, to an often-encountered situation in mathematics, an equation. By the end of the solution my gut says they will have found it praiseworthy, the standards of the OCC model.

The Theoretical Model

Following the advice of the distinguished teaching professor at SUNY Fredonia, Robert Rogers in a similar situation, I placed the theoretical model after the teaching episode. This journal is primarily destined to math teachers, people who make an effort on an everyday basis to enhance their teaching performance. In this direction they need to see the practical part that is going to a varying degree to attract their interest on the real teaching material. On the other side I could not avoid escorting the solutions to both parts of the exercise with comments that refer to the OCC model, especially when its application should clearly appear. The same model has appeared in the ICTM (Iowa Council of Teachers of Mathematics) journal and I decided to present the next paragraphs verbatim not to modify the essence of the aforementioned model (Petakos, Winter 2019-2020).

By referring briefly to the theory of emotion, we state here the underlying pillars of the emotion itself, which is goals, standards and tastes (Ortony & Clore, 2015). These three values, namely goals, standards and tastes, are quintessential of the emotion theory. Notwithstanding the fact that someone might have proceeded to the analysis of the teaching episode relying on the etymology per se of the above-mentioned values, I proceed with the next paragraph.

> Appraisals can therefore concern the *outcomes of events* evaluated as *desirable* (or not) in terms of goals, the *actions of agents* evaluated as *praiseworthy* or *blameworthy* relative to one or another kind of standard, or *the attributes of objects* evaluated as *appealing* (or not) as a function of one's tastes. These three sources of evaluation yield three kinds of affect that contribute to the distinctiveness of various classes of emotion, namely, being *pleased* or *displeased* about event outcomes, *approving* or *disapproving* of the actions of agents, and *liking* or *disliking* (the attributes of) objects. (Ortony & Clore, 2015, p.310)

Ortony et al (1988) in a worthy effort to cognitively analyze emotion emphasize on the three aforementioned pillars, goals, standards and tastes, using respectively the concepts *events* (goals), *agents* (standards) and *objects* (tastes).

> When one focuses on events one does so because one is interested in their consequences, when one focuses on agents, one does so because of their actions, and when one focuses on objects, one is interested in certain aspects or imputed properties of them *qua* objects. (Ortony et al, 1988, p.18)

To finally dilute any misgivings about the concepts discussed above (Ortony et al, 1988) we state a paragraph taken from a mathematical article exemplifying them:

- *Objects.* Emotions resulting from reactions to objects 'qua' objects (attraction emotions) are all variations of the affective reactions of liking and disliking (typical examples are love and hate)

- *Events.* This is the class of affective reactions of being pleased and displeased. These affective reactions arise when a person construes the consequences of an event as being desirable or undesirable (typical emotions are joy, hope, fear)
- *Agents.* Affective reactions of approving and disapproving (typical emotions are pride, shame, admiration, reproach). (DiMartino and Zan, 2011, p 474).

Even partial application of the aforementioned OCC model can really contribute to making mathematics flourish not just between students and teachers but also among math teachers dedicated to their educational mission, sparking a fruitful dialogue that will prove beneficial to all involving sides.

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