Utah Mathematics Teacher

Utah Council of Teachers of Mathematics

"Exploration"

Editors:

Alee Lee
aleeslee@weber.edu

Danielle Divis
danielledivis21@gmail.com

JOURNAL EDITORS

Alees Lee, Ph.D.
Weber State University
801-626-7105
aleeslee@weber.edu

Danielle Divis, Ph.D.
Herriman, UT 84096
801-694-7295
danielledivis21@gmail.com

UCTM 2022 BOARD

CALL FOR ARTICLES

The Utah Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Utah. All are encouraged to contribute articles and opinions for any section of the journal. Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to Alees Lee at aleeslee@weber.edu or Danielle Divis at danielledivis21@gmail.com. A cover letter containing author's name, address, affiliations, phone, e-mail address and the article's intended audience should be included.

UTAH MATHEMATICS TEACHER

Volume 15, Winter 2023

"Exploration"

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UCTM President's Message

Andrew Glaze, Ph.D.

Recently I was reflecting on the wonderful and varied avenues that people take to enter the teaching profession. While many of us (myself included) entered the profession after formal educational training at a university, others are increasingly coming to the field as a career change. I absolutely love the culture that those previous and varied experiences bring to a community of learners. I have personally worked with former military leaders, engineers, writers, actors, and professional athletes.

Whatever route you took to enter the classroom, thank you for coming. Teaching is not an easy job. Added to the seemingly endless list of lessons to plan and tests to grade are a litany of other tasks which necessarily accompany this job. There is lunch duty, playground duty, bus duty, club supervision, sports coaching, committee obligations etc. If you are anything like me, your students and your lessons are often among the last things you think about before you go to bed at night and the first things you think about when you rise in the morning. This is a demanding profession.

At the same time, this is also a very satisfying profession. Watching a student gain confidence because, with your encouragement or help, they were able to do something very hard for the first time in their lives is a moment that I hope everybody in this field experiences many times over. Having a former student contact you to say "TThank you. You made a difference" is a heartwarming moment only earned after countless hours of dedicated service.

Often our days are so busy that we lack time to thoroughly collaborate or communicate with our fellow teachers. That's part of why attending conferences and reading journals is so important. The members of the UCTM governing board are continually trying to bring you opportunities to reflect, connect, and refresh. It is my hope that you will find something inside this journal or at our accompanying conference that inspires you. Maybe it sparks a thought of something you also would like to share. If so, please submit your own journal article or presentation proposal.

Thank you again for your service.

Letter from the Editor

Alees Lee, Ph.D.
Danielle Divis, Ph.D.

The editors of UCTM's 2023 Journal are absolutely thrilled to bring you the first ever Winter publication. The release of this publication coincides with UCTM's 2023 Conference, and we are delighted to have several articles that discuss equity in education as well as the conference theme "Explore".

Reflecting on the conference theme in reference to my own instruction has brought to light the role of exploration in the growth of not only my students but also my own teaching. I, Dr. Alees Lee, am an assistant professor of mathematics education at Weber State University. In this role, I have the privilege to teach content courses for both elementary and secondary preservice teachers. As I facilitate their exploration of mathematical tasks in my classroom, I am humbled by their ingenuity, perseverance, and most of all their curiosity. My most memorable, uplifting teaching moments involved seeing students exhibit those traits, and I realized those moments occur more often when students are provided the opportunity to explore. So, as we continue through this school year, I hope you will join me in constantly asking yourself "how will my students explore mathematics today?"

Within this journal, you will find articles that encourage the exploration of your teaching profession. Of particular note is Michelle Parslow's article offering her story of allowing students to explore mathematics through the making, launching and mathematically modeling of rockets. She also provides lesson resources for engaging your students in this task. Next up, are two articles that offer strategies for planning instruction that can lead to student exploration of mathematics. The first of these articles, *Monitoring Charts: A powerful tool for planning and gather data,* provides insights into the many benefits of using a monitoring chart to plan and structure instruction, while the second article by Shannon Olson articulates the importance of using learning trajectories and progressions to provide meaningful, scaffolded opportunities for students to learn mathematics.

In addition to the above articles, we are excited to present three submissions on equity. In his article, *What's going on with Critical Race Theory and why should a math teacher care?*, Trevor Warburton summarizes Critical Race Theory and its influence on education. At the end of his article he offers resources for consideration. Finally, to conclude our publication, Molly Basham and Camille Lund each offer reviews of equity focused books.

We hope you enjoy the ideas offered up in this journal and hope that you take the opportunity to explore your instruction. In addition, please consider submitting your own articles or serving as a reviewer for further publications.

Note. Any mistakes are the sole responsibility of the editor and will be remedied in the online journal. Please send corrections to aleeslee@weber.edu or danielledivis21@gmail.com.

Corequisite Model in an Introductory Statistics Class

Jameson C. Hardy (Utah Tech University)

Mathematics and data analysis are becoming increasingly important for students entering the workplace. Most degrees require a course in mathematics; and students who enroll in a mathematics course during their first year of college triple their chances of completing the general mathematics requirements in Utah (Utah System of Higher Education, 2015). Often, mathematics classes become a roadblock for students as they work towards their degree because they are not college ready. One solution is to offer students remedial mathematics courses. When taken in the first year, Utah students requiring remediation have an equivalent rate of degree completion compared to students not requiring remediation (Utah System of Higher Education, 2015). The disadvantage of this approach is the increased cost for students needing to take extra classes. An alternative approach is to offer general education (GE) mathematics classes using a corequisite model. Corequisite is defined as "a formal course of study required to be taken simultaneously with another" (Merriam-Webster, 2021). The corequisite model suggests removing the remedial classes and instead providing students with extra support to fill holes in their understanding (Beamer, 2020). This paper explores the implementation of a curriculum designed with the corequisite model in mind.

Context of Mathematics Courses at Utah Tech University

Utah Tech University (formerly Dixie State University) is a small university serving approximately 9000 students in southern Utah. Utah Tech's student population consists of 53% female and 47% male, and 22% of students belong to a minority group. Utah Tech is an

open-enrollment institution, meaning that the school has a 100% acceptance rate for students that apply (Utah Tech University, 2021). This poses unique challenges for the university, one of which is a high demand for remedial mathematics courses.

In my personal teaching experience, students that take remedial mathematics courses have struggled with the concepts for a very long time. Many of them enter the courses with negative attitudes about their abilities in mathematics. The attrition rates of the remedial courses at Utah Tech are quite high. For example, in the 2017-2018 school year, 47% of students enrolled in the intermediate algebra class failed to progress to the next mathematics class. Anecdotally, I observed that if the student completed a remedial course, the student would pass. Something was causing students to stop participating in the courses, and it caused members of the mathematics department to explore the situation.

Formation of the Math Pathways Committee

Three of my coworkers and I saw problems within our remedial courses and set out to fix them. We formed the Math Pathways Committee in the fall of 2018 with the goal of improving the passing rates of the students in our remedial mathematics courses. The committee started as lunch discussions among like-minded individuals and has since become a recognized committee on campus. The committee consists of two mathematics education specialists, and two pure mathematicians. Three of the members are coordinators for our remedial and general education courses, and one is the coordinator for our concurrent enrollment program.

Improving passing rates decreases the frequency of students retaking courses, which translates into less time spent before students begin their general education (GE) mathematics courses. We decided improving the passing rates was the logical place to start. We knew that doing so would be a multi-year process and required several stages. The phases we identified

were (1) work with advisors for better placement of students into the proper courses, (2) create a non-STEM pathway, and (3) address the rising trend of corequisite models.

Phase 1: Better Placement

The first phase of improvement focused on the placement of our students. One of the committee's hypotheses was that students failed in Math 1010 because they were ill prepared for the class and/or should be in the previous course. Before the fall of 2018, we offered two remedial courses, beginning algebra, titled Math 0900: Transitional Math I, and intermediate algebra, titled Math 1010: Transitional Math II. Students with ACT scores below 12 placed into Math 0900 and students with ACT scores between 12 and 23 placed into Math 1010. Math 1010 had a high attrition rate and anywhere between 40-60% of students failed to progress to the next class.

Upon investigation, we found that Utah Tech University was placing students in Math 1010 with far lower ACT scores than other Utah institutions. We also found that 80% of students that failed to progress from Math 1010 were placed with an ACT score of 17 or below. This indicated to the committee that the ACT scores the university used to place students were hindering many of our students, and that we needed to rethink the cutoff scores for placement. We changed our cutoff score for Math 1010 to 18 to better match other institutions in Utah. This created a new challenge. Students with ACT scores of 12-18 now had to complete two remedial courses instead of one. We addressed this issue with another option, which we called the non-STEM pathway.

Phase 2: Non-STEM Pathway

In our second phase of improvement, we developed a new class trajectory for non-STEM majors as seen in Figure 1. In Figure 1, remedial courses are indicated in gray. The left path,

highlighted in blue, shows the course sequence for STEM majors such as engineering, mathematics, and computer science. Students have the option to take College Algebra (Math 1050) and Trigonometry (Math 1060) separately, or as a combined class (Math 1080). The goal of the blue pathway is to serve students that need Calculus I (Math 1210) for their degree. The middle pathway, highlighted in yellow, is for students with business or technology majors. The last mathematics class required for their major is Business Calculus (Math 1100). The two pathways on the right serve students in non-STEM majors and depending on their major they can take Quantitative Reasoning (Math 1030) or Introduction to Statistics (Math 1040).

Figure 1

The Pathways for Students Taking Mathematics Classes at Utah Tech University (Formerly Dixie State University)
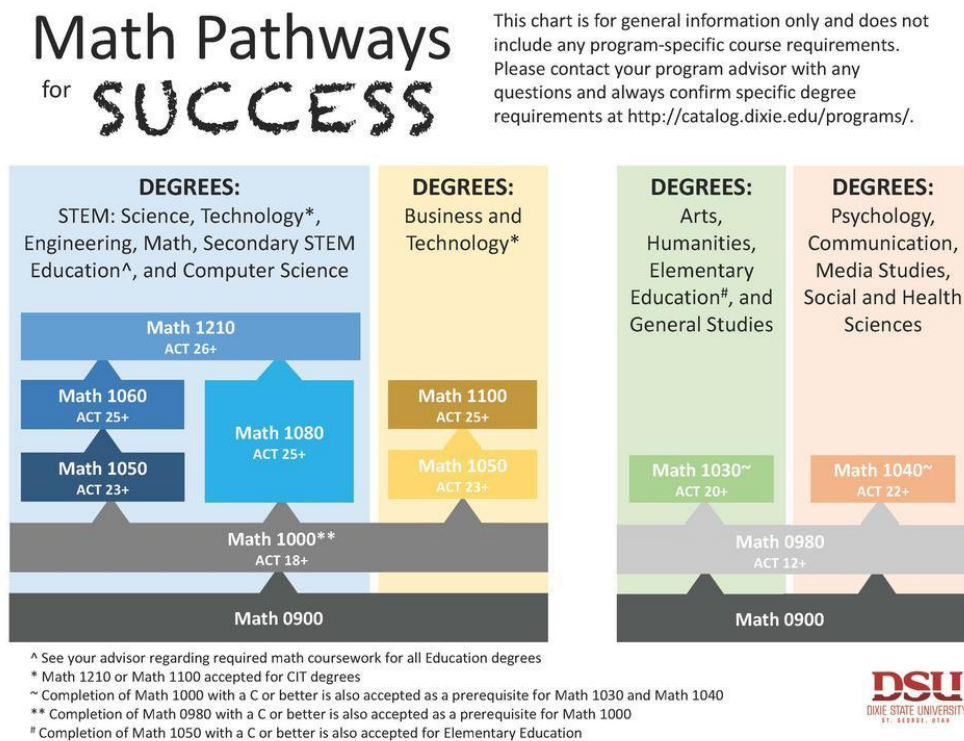
Note. The mathematics course sequence that students take at Utah Tech University depends on the student's major. Placement in the sequence is decided by ACT score. At the time this graphic was made the intermediate algebra course was known as Math 1000. It is now Math 1010.

Before the fall of 2018 the intermediate algebra course, Math 1010: Transitional Math II served as a prerequisite for all our general education mathematics classes. We designed Math 1010 to prepare students for College Algebra (Math 1050), but it was also a prerequisite for our Introduction to Statistics course (Math 1040). There was very little in Math 1010 that prepared students for Math 1040 and was preventing many students from progressing. The committee proposed a new pathway, highlighted in green and salmon in Figure 1, which included a new course called Transitional Math IIB (Math 0980). This course can be seen in Figure 1 highlighted in light gray. This course focuses on quantitative reasoning and statistics. Introducing Math 0980 allowed Math 1010 to remain rigorous for the STEM majors and allowed us to target preparation for Math 1040 to only the students that needed it. The passing rates of all our classes increased and the change received positive feedback from administration. Many advisors experienced confusion with the new pathway; however, we held several individual meetings with them to help them understand our new goals.

Phase 3: Corequisite on the Horizon

Fewer students retaking classes is positive, but there is still the problem of students not starting in their GE classes. This amounts to a substantial amount of time, money, and resources spent by students and the university before the students are ready to take mathematics classes that count for graduation. Universities across Utah are addressing this issue with a corequisite model. Corequisite is defined as a formal course of study required to be taken simultaneously with another (Merriam-Webster, 2021).

The corequisite model suggests removing the remedial classes and teaching students pre-requisite knowledge as it is needed (Beamer, 2020). We have been resistant to implement a corequisite course in the mathematics department at Utah Tech because we have not determined that the current research is conclusive enough. For example, Logue et al. (2016) conducted a randomized controlled study in which students needing remediation were assigned to an elementary algebra course, an elementary algebra class with a workshop, or an elementary statistics course with corequisite support. The results showed that 56% of the students passed the elementary statistics course, and only 39% of students passed the elementary algebra course. The addition of the workshop in the algebra course resulted in 45% of students passing. The results seem impressive; however, comparing a statistics course and an algebra course is problematic for the Math Pathways Committee. Despite our reservations, we may not have a choice for much longer. The chair of the mathematics department recently informed us that Utah Tech is the only university in Utah not currently using a corequisite model in some form; there has been a considerable push at our university to implement corequisite courses. We felt that if forced to try a corequisite model, we want to do it on our terms.

The Math 1040 Corequisite Curriculum

The Math Pathways Committee decided that the best place to begin exploring a corequisite model would be in our Introduction to Statistics (Math 1040) course. Math 1040 begins with descriptive statistics, such as measures of central tendency. Next, several chapters are dedicated to probability topics such as the addition and multiplication rules, complements, and the binomial distribution. Lastly, we cover inferential statistics such as confidence intervals, hypothesis testing, and ANOVA tables. We decided that we could teach any algebra concepts as needed, such as the slope-intercept form of lines; and we could supplement any statistics

concepts that are difficult for lower-achieving students. Our long-term goals are to (1) take advantage of a university grant to offer supplemental instruction via the campus tutoring center, (2) develop materials to begin offering supplemental instruction ourselves, and (3) design a corequisite course using previously developed materials. We are currently in the preliminary stage.

Preliminary Supplementary Instruction Implementation of the Curriculum

Fall of 2021, the campus tutoring center obtained a grant to offer supplemental instruction (SI) on campus. Any department can take advantage of the program. The grant pays a supplemental instruction leader to work under the course instructor. They are required to attend class, hold office hours for students, and hold an extra class session each week. Math 1040 is a three-credit course and is predominantly lecture. Students that enroll in the SI class receive an additional credit to attend a fourth-class period. The three normal meeting times are identical to a traditional Math 1040 and are run by the instructor, whereas the SI leader runs the SI session.

The SI session's goal is not to work on homework. Instead, students complete assignments intended to help conceptualize topics from the current week. Some assignments require using concepts learned on real data. For example, after learning measures of central tendency, students found the mean scores of a previous semester's test results. Other assignments are hands-on activities intended to provide a physical representation of the concept. For example, when learning about the normal distribution, students weighed suckers to see if the distribution of weights was normal. Assignments are only graded for completion, but student answers are often used during normal class discussions.

Student Results

The probability unit informs the inferential statistics and gives context to the normal

distribution. Historically, the probability sections are the hardest for students to grasp, and therefore I will exhibit these sections specifically. The first probability assignment's goals were (1) to highlight the difference between theoretical probabilities and experimental probabilities, and (2) to help students conceptualize the law of large numbers. Students calculated basic probabilities, such as the probability of rolling a one on a die, and then rolled a die 100 times to compare the results. The last question asked for any patterns noticed by the students. Figure 2 shows student answers to the question. As seen in Figure 2, many students noticed that the experimental probabilities were close to the theoretical probabilities but failed to explain the effect of the law of large numbers effectively. In the previous class session, the law of large numbers was defined, "As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome" (Sullivan, 2021, p.252).

Figure 2

Student Responses Concerning the Law of Large Numbers

15. Describe any patterns you notice.

- experimental probability is usually w/in .05 of the theoretical probability
- as you add more outcomes the probability is higher & more accurate

15. Describe any patterns you notice.

- THE EXPERIMENTAL PROBABILITY IS SLIGHTLY OFF EACH TIME (EITHER HIGHER OR LOWER)

- THE EXPERIMENTAL PROBABILITY WAS CLOSER TO THE THEORETICAL PROBABILITY WHEN YOU COMBINED ALL THE GROUPS RESULTS

Note. This figure shows two student responses comparing probabilities calculated and probabilities found experimentally. The first is partially correct and the second is fully correct.

The second student in Figure 2 effectively summarized the law of large numbers in her own words by explaining that as the number of observations increases, the experimental probability gets closer to theoretical. The first student was mistaken by saying, "The probability is higher." This is important to note because I highlighted the law of large numbers in class the next day, using the assignment as a formative assessment.

The next assignment I wish to highlight covered the multiplication rule of dependent events. I taught the topic the day before, and now students practiced what they saw in the SI session. Students calculated various probabilities, such as the probability of drawing two red cards from a standard deck, and then performed the experiment to compare. The goal of the

assignment was to highlight the difference between specifying the order of events or not. For example, one question asked for the probability of rolling a 1 and then a 4 on a die. This is calculated using the multiplication rule.

$$P(1 \text{ and } 4) = P(1) \cdot P(1) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$$

Order is inherent in the calculation. One calculates the probability of rolling a 4 and then a 1 the same way. Students struggle when the question does not specify order. For example, the probability of rolling a 1 and a 4 with two rolls is $\frac{2}{36}$ because the event can occur as 1 and then a 4 or a 4 and then a 1. One must find each individual probability and then add them.

Students also struggle with the related concept of "at least one." For example, to find the probability of drawing two cards and at least one of them is a club one could consider each way the event could happen, calculate the probabilities, and add them. Alternatively, one could calculate the probability of not drawing a club at all, known as the complement. The probability of this complement, when subtracted from one, gives the desired probability. I asked students to consider the complement of drawing two kings. Figure 3 shows an incorrect response and a correct response to the question. The second student understood that she needed to calculate the probability of drawing two kings without replacement and then find the complement. The first student understood which events make the complement, as indicated in the sample space, and understood that with a complement a probability must be subtracted from one, however, she struggled with the multiplication rule. The student calculated the probability of drawing only one king and subtracted that probability from one. This seems to indicate the student has not conceptualized compound events. Again, I used the responses as a formative assessment and discussed compound events with my students again.

Figure 3

Student Responses Concerning Complements of Compound Events

17. **Concept Check:** Imagine you draw two cards, without replacement. What is the complement of drawing two kings?

KK   4/52   .076

K?
?K
??

.92

17. **Concept Check:** Imagine you draw two cards, without replacement. What is the complement of drawing two kings?

KK
K?
?K
??

$\frac{4}{52} \cdot \frac{3}{51} = .0045$

$1 - .0045 = \boxed{.995}$

Note. The first student's response is incorrect, and the second student's response is correct.

Challenges to Address for the Future

One challenge that needs addressing in the future is the SI leader was a Utah Tech student that previously received an A in the course. The student may lack Mathematical Knowledge for Teaching (MKT). Their mathematics background may not be as strong as many professors desire, or they may fail to interpret something in context. This challenge is clear when looking at content taught by the SI leader. For example, in Figure 4 students compared the experimental results with the calculated results of rolling dice. The goal was to allow students to conceptualize what the area under a normal distribution is representing. I hoped they would see that most sample means of the dice rolls performed were close to the mean of the random variable, and not many were far away.

Figure 4

Incorrect Student Response Influenced by Supplemental Instruction Leader

The Experiment:

Now we're ready to test these probabilities experimentally. With your partner, roll a die 30 times and calculate the mean. Repeat this as many times as you can in the allotted time (There is a table provided to help with data collection). Provide the SI leader with your means and use the combined data to answer the questions below.

3.9878

0.3439

4) What is the experimental probability of getting an average below 3?

.0780

5) What is the experimental probability of getting an average above 3.8?

.182

Note. The SI leader misunderstood the question and gave students incorrect answers.

The SI leader failed to highlight how many sample means were close to the expected value of 3.5 and instead instructed students to put the sample mean and standard deviation into a normal calculator. Furthermore, the SI leader was unable to help students with the homework from this section. Part of the cause of the incorrect answer on the assignment is the question's wording. I should have written something like, "What proportion of means were below 3?", however, the SI leader could not answer most of the questions from the homework, which indicates a lack of understanding of the topic.

The goals of each activity need to be explained to the SI leader, preferably in the form of a study plan. I naively assumed the SI leader would understand the goals because she went through the course herself. I should have explained the goals of the course better. If the SI leader is available to run the sessions for several semesters, they will learn the material at an appropriate level. The challenge reappears, however, when a new SI leader is required.

Alternatively, mathematics and mathematics education majors have higher content knowledge and therefore may serve as better leaders because of their math background; however, these students do not take Math 1040, which is a new issue.

Conclusion

I was pleased with the formative nature of the SI sessions and feel that it has been successful in helping students conceptualize statistics concepts; however, I believe that an expert needs to be in the room during the SI sessions for it to be the most effective. Our current plan at Utah Tech is to pay an instructor to run the extra class times. Tentatively, the course will be a three-credit class, and students below a certain threshold will be required to attend two more sessions. One session will cover prerequisite knowledge and offer homework help. The other session will provide activities and supplemental work to solidify the concepts that week. This plan may change as we further explore the corequisite model. It is my hope that our corequisite plan, in conjunction with our new pathway, will keep costs for students low with fewer classes to take, but it will also help students succeed in their mathematics classes without lowering the quality and expectations of our courses.

References

Beamer, Z. (2020). Mathematics corequisite remediation and direct enrollment: Addressing

misconceptions and concerns. *Inquiry: The Journal of the Virginia Community Colleges,*

23(1), 1-13. https://commons.vccs.edu/inquiry/vol23/iss1/9

Utah Tech University. About. (n.d.). Retrieved November 25, 2021, from

https://about.utahtech.edu/

Logue, A. W., Watanabe-Rose, M., & Douglas, D. (2016). Should students assessed as needing

remedial mathematics take college-level quantitative courses instead? A randomized

controlled trial. *Educational Evaluation and Policy Analysis, 38*(3), 578-598.

https://doi.org/10.3102/0162373716649056

Merriam-Webster. (n.d.). Corequisite. In Merriam-Webster.com dictionary. Retrieved November

25, 2021, from https://www.merriam-webster.com/dictionary/corequisite

Sullivan, M. (2021). *Statistics: Informed decisions using data.* (6th ed.). Pearson.

Utah System of Higher Education. (2015, August 11). Math and graduation: High school math &

first year of college critical to completion.

https://ushe.edu/high-school-math-critical-to-completion/

Integrated STEAM Learning Task: Make, Launch, and Mathematically Model Rockets with Quadratic Equations

By Michelle Parslow (Wahlquist Junior High School)

Abstract

This article describes a project I facilitate in a Science, Technology, Engineering, Art, and Math (STEAM) projects class. The purpose of this lesson was to help students answer some of the "why" of mathematics and connect mathematics in an interdisciplinary way in order to learn standards relating to the STEAM subjects using the Hearts-on, Hands-on, Heads-on (3-H) learning cycle (Sackes et al., 2019; Trundle & Smith, 2017). During the lesson, students learn about designing, building, launching, and mathematically modeling compressed air rockets. This article includes a materials list, connections to technology resources, and some examples of students' work and experiences. It also describes how to teach the STEAM rocket lesson and includes students' reflections on their experiences and what they learned. I hope that you and your students will enjoy and learn as much as we do.

Keywords: STEAM education, integrated learning, rockets, mathematics, grades 6-12

Integrated STEAM Learning Task: Make, Launch, and Mathematically Model

Rockets with Quadratic Equations

This article describes a lesson plan conducted in a Science, Technology, Engineering, Art, and Math (STEAM) projects class. The purpose of this lesson plan was to use STEAM-integrated teaching and learning to enhance mathematical connections and understanding, along with answering the "why" that students often ask when learning mathematics.

The "why" in mathematics teaching and learning is a question that comes from a desire to understand the connections that mathematics can provide between subjects and to real-world activities. I posit that it can best be addressed through an integration of Science, Technology, Engineering, Art, and Mathematics (STEAM) learning activities. Yakman's (2008) definition of STEAM education in her dissertation was: "ST∑@M is a developing educational model of how the traditional academic subjects (silos) of science, technology, engineering, arts, and mathematics can be structured into a framework by which to plan integrative curricula" (Yakman, 2008, p. 1). She then concludes with "STEAM Education: Science and Technology, interpreted through Engineering and the Arts, all based in a language of Mathematics" (p. 18). STEAM projects facilitate students learning like scientists (Thuneberg et al., 2018), provide spaces to do more inquiry and hands-on learning, and develop spatial reasoning. Integrating STEAM subjects improves attitudes and promotes motivation and engagement to learn mathematics. This integration provides advantages to diverse learners who experience positive effects (Brigham et al., 2011; Cunnington et al., 2014; DeJarnette, 2018; Grouws et al., 2013; McCarthy, 2005; Salami et al., 2017; Thuneberg et al., 2018).

The objectives of this lesson include visual arts standards 7-8.V.CR.2 to document the creative process in new media and 7-8.V.CR.11 to reflect, explain, and plan revisions. The Common Core State Standards in Mathematics (CCSS-M) include interpreting and modeling with linear, quadratic, and exponential functions in 8th grade through high school. Using graphing utilities and computer systems to model is encouraged in relation to other standards (Common Core State Standards, 2010). The uses and connections of mathematical representations are also one of NCTM's eight Mathematics Teaching Practices (NCTM, 2014).

This lesson integrates the STEAM subjects in a paper rocket-building and compressed air launching project that I teach in my STEAM projects course. My positions in teaching have been in grades 6 to 9 over the course of 28 years including the subjects Math 7, Algebra A and B, Algebra 1, Geometry, Algebra 2, Math Course 1 (CCSS), Art 1, Art 2, Art 3 and STEAM projects. I designed this unit hoping to address motivation, diverse learning needs, and integration to address the "why" of math. I envision the STEAM projects course as an extension or supplement to traditional mathematics courses in the middle or high school grades. I hope you and your students enjoy this project as much as my students and I do at the junior high school where I teach. This course is open enrollment for our 7th to 9th grade students with no previous experience required. The structure of the course is inquiry and project-based learning. I even welcome the life-skills class students to join in with a peer tutor to help them. I believe in the opportunity to learn for all students.

Materials and Preparation

This rocket-building and mathematics modeling project is one of the students' favorites. The recommended materials needed for a class of approximately 36 are included in Table 1. I plan for this module to take about 2 weeks to complete.

Table 1

Materials for the Rocket Module

Material description	Quantity
A compressed air rocket launcher made with PVC pipe	1
An air compressor	1
Extra PVC pipes 1 foot long for students to use as molds to create their rockets the right diameter for the launcher.	12
Cardstock or magazine paper (thick)	40 sheets
Compasses	36
Scissors	36
Rulers	36
Packing tape	12
Electrical tape	24
Paint, colored pencils, or markers for aesthetic design	Red, yellow, blue, white, and black
A yardstick for students to calibrate their stride to 3 feet.	1
Hot glue gun and glue	1 gun, 100 sticks of glue

Along with the materials listed in Table 1, students will need to have access to Chromebooks or other technology that can support Tinkercad (https://www.tinkercad.com/) and Desmos (https://www.desmos.com/calculator). The primary paint colors are recommended because all of the other colors can be mixed from these. This list is not exhaustive or the only materials that will work. You can make substitutions based on your available resources. The internet is full of designs and material resources.

Figure 1

Compressed Air Rocket Launcher and Yardstick Setup for Launching Day

Figure 1 shows the compressed air rocket launcher, a student's rocket ready to launch, a yardstick for calibration of student steps, and the launch area. Recently we have re-designed the launder to replace the handle with a push button and sprinkler valve. The release happens faster with this new design and the rockets are launching farther. Examples of compressed air rocket launchers can be found on the internet (e.g.,

https://www.instructables.com/Compressed-Air-Rocket-Launcher/, and How to Make an Air-Powered Rocket Launcher | I Like To Make Stuff)

Hearts-On, Hands-On, Heads-On

I use the Hearts-On, Hands-On, Heads-On model (3-H model) of learning, which includes playing with incidental learning, nurturing curiosity, intentional learning, applying learning, discussing, and reflection (Sackes et al., 2019; Trundle & Smith, 2017). This is recently being adapted for use with older students as well. The 3-H model for learning goes well with STEAM projects as the Hearts-on often incorporates the play, creativity, and curiosity associated

with both art and science. The Hands-on often incorporates the project-based, experimental part. Heads-on can be the mathematical modeling, discussion, and reflections about the learning.

Hearts-On

The students learn about how rockets work by researching on the internet, following their curiosities, and watching my preselected resources. Students have a natural desire to investigate rockets.

Rocket Investigations

I have students research and investigate the role of fins, aerodynamics, and weight distribution in rocket science. I post YouTube videos explaining the function and purpose of the parts of a compressed air rocket in my Learning Management System (LMS). They love planning their own rocket, building it out of paper, then shooting off their rockets with compressed air. They explore functions and write a function that models their rocket launch to include in their reflection.

After completing their research, students create a plan for the materials, dimensions, and designs. They draw it up first on paper, and second, they create a model on Tinkercad (https://www.tinkercad.com/dashboard). The example in Figure 2 includes a launcher, however, I do not require the launcher to be part of their Tinkercad-built model. A scaffolded lesson I provide for using Tinkercad is creating a rocket in Tinkercad in our LMS, we use Canvas, in addition to class explanations/demonstrations. They use their designs of a rocket (on paper and on Tinkercad) that will be propelled by compressed air and built out of cardstock to gather their materials. I have pieces of PVC pipe that are the same size as the PVC on the end of the rocket launcher that the students can use as a form. I have a PVC pipe rocket launcher that is connected

to an air compressor that was built by my father-in-law. There are many resources online for building compressed-air rocket launchers.

Figure 2

A Student's Tinkercad 3-D Model of Their Rocket
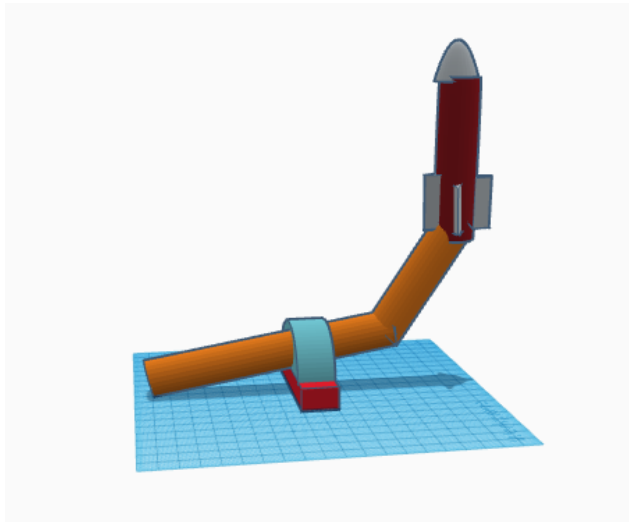
The student who created this Tinkercad model in Figure 2 was not required to include part of the launcher but choose to. I typically give a student some extra credit for going beyond my requirements.

Hands-on

The students build their rockets with cardstock and/or parts of magazines (Figure 3). I also provide packing tape, electrical tape, and compasses to draw the circles needed to create the cone. After the students have researched, designed, and built their rockets we chose a launching day based on weather reports and the school schedule.

Figure 3

A Student with the Rocket They Built

In Figure 3 the student used cardstock, electrical tape, packing tape, and PVC form pipe to build this rocket. She got the design from watching videos I included in a Canvas module on the purpose of some of the various parts of a rocket, such as fins, the cone, balance, and her own creativity and plan. One tip that I give my students is to tape across the top of the main body to seal in the air, the cone does not usually seal well enough to hold in the air pressure.

Launching Their Rocket

Checking the weather and choosing the launching day creates great anticipation. We do it outside toward an open area. The students check the air pressure and pull the handle to launch their rocket. They then count their strides to the final resting place of the rocket as the distances are usually too far to easily measure other ways (most go between 50 and 300 feet). Students collect data from their launch including video, the distance their rocket went (stepped off after calibrating their stride to 3 feet with a yardstick, then multiplying their strides by 3 to give the distance in feet), and the psi of the air at the time of their rocket launch. See examples of a student launching their rocket here and here. Some comments about shooting off the rockets

include "I really enjoyed shooting rockets because I got to experience something new" and "Shooting off my rocket was interesting because of the way my rocket was built."

Heads-on

*Reporting Their Data and Experience in an Online Discussion*

I demonstrate to the students how to reply, download, upload, and post their videos and how to describe a rocket launch in a Canvas discussion. Students post videos of their rocket launch, an explanation of their experience, and the distance their rocket travels to the class discussion.

Mathematical Modeling of the Path of Their Rocket

When we return to the classroom, the students explore an equation graphing program, see Figure 4 created in DESMOS (https://www.desmos.com/calculator) (DESMOS, 2019). I include how-to videos that have the purpose to help them navigate DESMOS. Students graph several types of equations to inductively discover the relationships of the variables, coefficients, and the shape of the graphs (cycling back to Hearts-on).

Figure 4

Screenshots of a Student's Equation/Graph Explorations
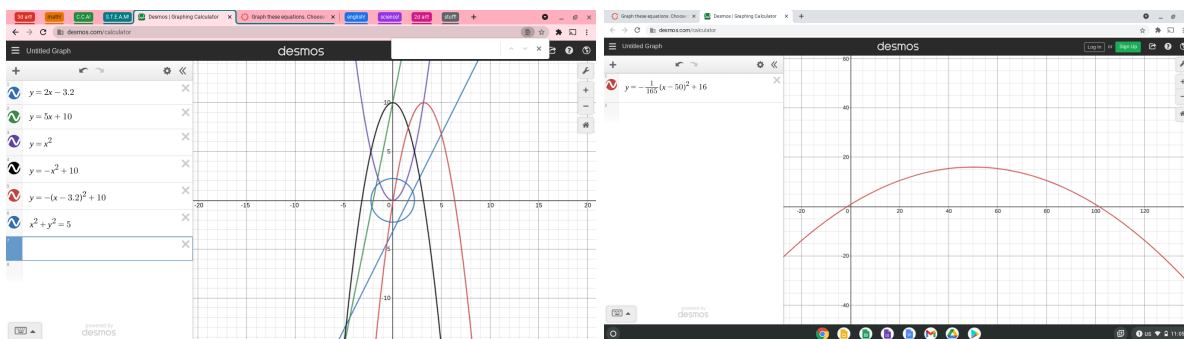
After students explore different forms of equations (Figure 4) I ask them to tell me which equation's graph most closely models a path a rocket might take. I have them analyze the

equation that produced that graph and explore what parts (variables, coefficients, exponents) make the graph do translations left or right, or up or down, how the coefficients affect the steepness or flip it from opening up to opening down. Then I ask them to construct an equation that models the path that their rocket took during the launch (matching the x-intercept to the distance in feet and the graph's height to the approximate height that they saw their rocket go). I check with students as I walk around and answer questions and provide additional scaffolding as needed. I then do an in-class discussion with the students showing DESMOS on my class screen, and asking students to participate in telling us how to create an equation that could be a model for one of our rocket launches. With all this scaffolding and exploration, students usually feel empowered to create equations that graphically represent their rocket's launch path, see this student submission sample. This activity connects mathematics to their experiences so their "why" is anchored whenever they work with equations and their graphs.

The students then write up a reflection on the process, noting what they learned, the equation that best models their rockets' path, and what they would improve for another design and launch of a rocket.

Student and Teacher Reflections

I require students to write about the process they went through and reflect on it, including a video of their launch, and their collected data. I use the following guiding questions: How do compressed air rockets work and how do the function of the main parts (e.g. fins, cone)? What went well in building your rocket and what could be improved? What data did you collect from your rocket launch? (show and tell me about it). What explorations with function graphing did you do and which equation that you created best models the path of your rocket launch? (Include a screenshot of the Desmos graph of your model equation). What did you learn in the process

and what you would change if you did it again? After you write up everything you have learned so far, explain what more you want to learn about rockets, Tinkercad, equations, graphs, or mathematical modeling. Here are parts of some responses showing their learning and enthusiasm posted by my 7th and 8th-grade students:

> My experience with launching model rockets was good. It taught me a lot about rockets, Tinker Cad, and Desmos. It also taught me about equations, fins, and cones. For example, I had never used Desmos before and now I would consider myself fairly good at using it and I am confident with it. Another example is Tinker Cad. ...I made sure to watch lots of instructional videos on the topic. It made me a lot more confident and I was able to easily create a pretty cool-looking rocket. I also learned from launching both of my rockets, that lighter rockets tend to go quite a bit further than heavy rockets, my lighter rocket went 90 feet and my heavy rocket only went 53 feet. I made the one rocket heavier by adding more tape and hot glue and using bigger fins. On the topic of fins, my lighter rocket had small fins, was spinning the whole way, and was not very stable. But the heavier rocket with bigger fins was very stable in the air and stayed relatively straight. But, the heavier rocket did not go quite as far as the small rocket did. Modeling things such as rockets helps in the real world because you do not want to send a rocket into space that has not been tested, especially if it is carrying people. That could be a major problem. This is why we build models and test them to be safe and successful in the real world. Overall I had a very fun time building and launching the rockets.

And another excerpt from a different student:

> I learned that rockets need fins to fly straight or they won't fly straight. I also learned that if the rocket was too tight on the tube it would explode and if it was too loose it would

not go very far. Also if there was any spot where air came out it would not go very far either. Another thing I learned is how to use Desmos and it is really fun to play and explore with and find fun ways to use it. Also that there must be a cone for aerodynamics so it flies better.

And another:

My equation is $y=-1/14\ (x-16)^2+10$. It was kinda cool to see how you can graph any equation. I think I will use Desmos more often and that it will really help me in the future.

And another:

I did learn tho that your rocket has to be 100% airtight because if it's not the air will come out of the rocket during the launch, if the air does end up coming out then it won't all go into pushing the rocket into the air. Without the fins and the point (or the cone) your rocket will kinda just go everywhere where it wont go straight to keep track of. Instead it will kinda just be here and there. When making an air rocket you need to keep in mind that you have to make sure that everything is sealed off and that you have everything you need. Making my rocket i was trying to check that everything was sealed and that i had what i needed for the rocket to launch and i didn't really think about the size. So i learned that you don't just need to watch the big things you have to watch everything not just big things or the most important but everything from the body to the size everything plays a part in the final outcome.

And finally, this one:

I learned that you can't have your rocket too loose or too tight. If you have it too loose, it will not go as fast or as high as normal rockets do. If you have it too tight, it will explode when it launches. I also learned about how the fins are needed to keep the rocket going straight when launched. I learned a lot about the equations as well. I learned that the negative sign will put the graph on the opposite side of the graph. I learned that the rocket needs to be round because if it isn't, it will not go on the launcher correctly and if it doesn't go correctly, it will not launch and if it doesn't launch, it might explode. I also learned that to make a rocket, you need a replica to build the rocket on... I learned how to make the fins on Tinkercad. Another thing I learned is that Desmos taught me how to graph how a rocket flew. I also learned that you need a cone top because it will keep it going faster longer. I learned how to send a video to a computer by email. The next thing I learned is that it takes a lot of air pressure to shoot a rocket.

I love hearing from students about what they learn and enjoy doing during this project. There are so many reasons why this is a great learning experience for students.

Each time I teach this unit with students I reflect on the process, instructional resources I provide, the materials I provide, and the information in the student reflections. I evaluate if the students are able to successfully learn the objectives I have for this unit including the mathematics CCSS-M on graphing and modeling equations. I recently reviewed the CCSS-M and decided to add exponential equations to the investigation of multiple types of equations.

Summary

I do notice that I do not hear statements such as "Why do I have to learn this?" from students anytime in this lesson as I sometimes did when teaching stand-alone math lessons. The students are engaged and motivated, answer their own "why," and stretch themselves to do

"hard" things. Learning is not tested in the traditional sense but is evident in their submissions, reflections, and their writing of equations that can model an event. This rocket project addresses each of the STEAM subjects through the 3-H learning model. The science is addressed through research on how compressed air rockets work including the function of fins, cone, and balance. Students use technology to do the research, build a model in Tinkercad, take screenshots, create videos of their launch, explore equations and their graphs in Desmos, and connect to each other in Canvas discussions. Engineering is embedded in the design of their own paper plan, model rocket (Tinkercad), and building their 3-D launchable rocket. Art is integrated in creative in designs, making their models and launchable rocket aesthetically pleasing. In decorating their rocket, they also learn how to make any color from the primary colors. Math connections are using measurement, scale models, calibrations, and especially learning about the equations and graphs and using them to model the path of their personal rocket's path. Feel free to adjust the materials, scaffolding, and STEAM subject connections to best fit your situation.

References

Brigham, F. J., Scruggs, T. E., & Mastropieri, M. A. (2011). Science education and students with learning disabilities. *Learning Disabilities Research & Practice*, *26*(4), 223-232. https://doi.org/10.1111/j.1540-5826.2011.00343.x

Cunnington, M., Kantrowitz, A., Harnett, S., & Hill-Ries, A. (2014). Cultivating common ground: Integrating standards-based visual arts, math and literacy in high-poverty urban classrooms. *Journal for Learning through the Arts*, *10*(1), n1. https://doi.org/10.21977/D910119294

Dejarnette, N. K. (2018). Implementing STEAM in the early childhood classroom. *European Journal of STEM Education*, *3*(3), 18. https://doi.org/10.20897/ejsteme/3878

Desmos. 2019. "Desmos Graphing Calculator." https://www.desmos.com/calculator

Grouws, D. A., Tarr, J. E., Chávez, Ó., Sears, R., Soria, V. M., & Taylan, R. D. (2013). Curriculum and implementation effects on high school students' mathematics learning from curricula representing subject-specific and integrated content organizations. *Journal for Research in Mathematics Education*, *44*(2), 416-463. https://doi.org/10.5951/jresematheduc.44.2.0416

McCarthy, C. B. (2005). Effects of thematic‑based, hands‑on science teaching versus a textbook approach for students with disabilities. *Journal of Research in Science Teaching*, *42*(3), 245-263. https://doi.org/10.1002/tea.20057

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions; Ensuring Mathematical Success for All.* Reston, VA: NCTM. https://www.nctm.org/PtA/

National Governors Association Center for Best Practices (NGA Center) and Council of Chief

    State School Officers (CCSSO). 2010. *Common Core State Standards for Mathematics.*

    Washington, DC: NGA Center and CCSSO. http://www.corestandards.org/

Saçkes, M., Trundle, K. C., & Shaheen, M. (2019). The effect of balanced learning curriculum

    on young children's learning of science. *Early Childhood Education Journal*, *16*(2), 357-

    369. DOI 10.1007/s10643-019-00985-x

Salmi, H., Thuneberg, H., & Vainikainen, M. P. (2017). Learning with dinosaurs: a study on

    motivation, cognitive reasoning, and making observations. *International Journal of*

    Science Education, Part B, *7*(3), 203-218.

    https://doi.org/10.1080/21548455.2016.1200155

https://www.tinkercad.com/learn

Thuneberg, H. M., Salmi, H. S., & Bogner, F. X. (2018). How creativity, autonomy and visual

    reasoning contribute to cognitive learning in a STEAM hands-on inquiry-based math

    module. *Thinking Skills and Creativity*, *29*, 153-160.

    https://doi.org/10.1016/j.tsc.2018.07.003

Trundle, K. C. & Smith, M.M. (2017). A hearts-on, hands-on, minds-on model for preschool

    science learning. *Young Children*, *72*(1), 80-86. https://www.jstor.org/stable/90001494

Yakman, G. (2008). STEAM education: An overview of creating a model of integrative

    education.

Monitoring Charts: A Powerful Tool for Planning and Gathering Data

Nicole Gearing (Utah Valley University)
Andria Disney – (Utah Valley University)

Monitoring Charts: A Powerful Tool for Planning and Gathering Data

Monitoring charts are a valuable tool that mathematics teachers can use during the planning and implementation of their problem-based lessons. The chart supports planning by having teachers anticipate student strategies and possible misconceptions as well as pre-planning a variety of questions to ask students during the lesson. Moreover, the monitoring chart is a formative assessment tool that allows the teacher to capture data about individual student thinking, and it helps them plan a productive whole class discussion about the central math problem in the lesson. The monitoring chart can also assist teachers in making decisions about their next instructional steps and be a source for teacher self-reflection about the effectiveness of the lesson. In this article, we will share some research about mathematics teaching and assessment that relates to the use of monitoring charts, examine examples of monitoring charts and a vignette that demonstrates how to use one in a lesson, and explore the many benefits of using a monitoring chart as part of your mathematics teaching.

Background and Research

There are many benefits of using a monitoring chart in a math classroom. Before we explore those specifics, we will begin with some background information about the origin of monitoring charts as well as how they can be used for formative assessment and to promote equitable teaching practices.

Orchestrating Discourse

Facilitating productive discourse during a math lesson doesn't happen without proactive and purposeful planning by the teacher. Smith and Stein (2011) developed the *5 Practices for Orchestrating Productive Mathematics Discussions*, which supports teachers in this effort. The first phase requires teachers to ***anticipate*** student strategies as well as common misconceptions that might arise. Part of this process involves developing the monitoring chart to list those anticipated strategies and/or misconceptions and pre-planning questions for each. During the lesson, the teacher will ***monitor*** students as they solve the problem, recording what they notice in the monitoring chart and using the pre-planned questions to gain insight into student thinking. Then the teacher will use the data gathered in the monitoring chart to ***select*** students who will share their strategies and thinking during a whole class discussion and ***sequence*** the presenters to build from concrete representations to more abstract representations. Finally, the teacher will facilitate the whole class discussion with an emphasis on helping the students to ***make connections*** among the strategies and representations and with the mathematical goal of the lesson. Smith and Stein's practices can be integrated into any problem-based teaching approach, such as the Five Es (Engage-Explore-Explain-Elaborate-Evaluate) or Launch-Explore-Summarize (Wisconsin DPI, 2013).

Formative Assessment

Assessment is an integral part of the instructional cycle, and it is the place that we should begin when designing instruction. Using a backwards design model (Wiggins & McTighe, 2005), teachers should determine their learning goals first, then develop assessments that will gather data to determine the extent to which students meet those learning goals, and finally plan instructional activities that will help students successfully achieve the learning goals. Within this

framework, multiple and varied formative assessments should be utilized to monitor student progress toward the learning goals of the larger unit of instruction. Formative assessments should give teachers data/information upon which to make instructional decisions (Wiliam, 2015). In fact, this emphasis on formative assessment is reflected in NCTM's Effective Mathematics Teaching Practices (2014) where teachers should "use evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (p.53). The monitoring chart is a valuable formative assessment tool that teachers can use to monitor student progress, gain insight into students' mathematical thinking, and make decisions about how to facilitate the rest of the lesson as well as next instructional steps.

Equity Practices

It is imperative that all students have access to high-quality and engaging mathematics instruction and that teachers have high expectations for all of their students (NCTM, 2014). One way that teachers can promote equity practices in their classrooms is to design learning opportunities that have multiple entry points and allow for multiple pathways to a solution. Problem-based teaching creates space for students to select strategies or tools that make sense to them and pursue varied solution pathways, which are anticipated in a monitoring chart. Moreover, the monitoring chart requires teachers to pre-plan questions that prompt students to explain and justify their thinking. Bieda and Staples (2020) assert that mathematical justifications are an equity practice in mathematics teaching because they give students "***access*** to mathematical thinking and reasoning" and ***agency*** when students choose how to engage in the process of doing mathematics.

Monitoring Chart Vignette

Ms. Walters is planning a sixth-grade lesson on multiplying decimals using the standard algorithm (Utah Math Core Standard 6.NS.3). In this lesson, Ms. Walters planned to engage students by reading the picture book, *Six Dinner Sid* by Inga Moore. In the story, Sid, a cat, eats six meals a day. Ms. Walters wanted to focus on multiplying decimals, so she decided to pose a problem about the cost of feeding Sid: Imagine *Sid eats one can of food for each meal. Each can cost $.73. How much will it cost to feed Sid for one week?* Before beginning the lesson, Ms. Walters developed a monitoring chart to anticipate the strategies and errors students might use or encounter as they solved the problem (see Figure 1). The first strategy she thought students might use was to use the standard algorithm in parts, beginning by finding out how much it would cost to feed Sid for one day (6 x .73 = $4.38) and then multiplying to determine how much it would cost to feed Sid for the week (4.38 x 7 = $30.66). The second solution strategy Ms. Walters identified was determining the number of meals Sid would eat in a week and then multiplying that by the cost of each can (42 x .73 = $30.66). Even though Ms. Walters wanted to focus on the standard algorithm in this lesson, she knew that some of her students might still be using partial products, so she included this strategy as well. With her class in mind and the errors she's noticed in previous lessons, she included a space in her monitoring chart to identify students who are still struggling with their multiplication facts or making errors when they attempt to place the decimal when they finish multiplying. Next, she thought about questions she might ask students using each strategy. She wanted to ensure that she planned questions that would advance student thinking.

Figure 1

Feeding Sid Monitoring Chart

Strategy/Level	Questions to Ask	Students
Standard Algorithm in Parts 6 x .73 = $4.38 4.38 x 7 = $30.66	• What question am I answering when I find 6 x .73? • What does $4.38 represent? What do you need to do next? • Why is 6 x .73 less than $6? • How did you know where to put the decimal? Can you explain it to a friend?	Olivia Michael Charlotte James Mia Lucas Henry Ben
Standard Algorithm 42 x .73 = $30.66	• How did you know you should multiply 42 and .73? • How did you know where to put the decimal? Can you explain it to a friend?	Oliver William Liam Ava Sophia Isabella Noah
Partial Products |	40	2
.7	28	1.4
.03	1.2	.06		• How did you know you should multiply 42 and .73? • How did you know where to put the decimal? Can you explain it to a friend?	Levi Jack Ava Amelia Ezra Jayden
Made errors in placing the decimal when using the standard algorithm	• How did you know where to put the decimal? • Let's estimate the total. Does the placement of your decimal still make sense?	Jayden Noah Oliver Sophia
Made math fact errors	• The cash register doesn't agree with your total. Can you find your error?	Michael James Mia Lucas

Cannot Get Started	- How many days are in a week? - How can we figure out how much it costs to feed Sid for one day? - How many meals does Sid eat in a day? - How many meals does Sid eat in a week?	*Aurora* *Kai*

After launching the lesson, Ms. Walters monitored students as they began solving the problem. As she walked around the room, she used her monitoring chart to record the strategies she noticed students utilizing. As students worked, she asked them about their thinking using the questions on her chart. She then used the chart to purposefully identify students she wanted to share their work during the discussion portion of her lesson. While she was working with students, she noticed a few students still using the partial products strategy. She decided to have a student share this strategy first to ensure they begin to see connections between the partial products algorithm and the standard algorithm. She decided she would then have a student who broke the problem up by finding the cost of food per day and then per week share next, followed by a student who found the total number of meals and then multiplied that by the cost of a can of food. As students shared, Ms. Walters used her pre-planned questions from her monitoring chart to facilitate a discussion about the connections between the strategies students used and determining how to place the decimal point in the product.

At the end of the lesson, Ms. Walters collected the students' work and made sure she recorded each student's strategy and any misconceptions/errors they made in the corresponding boxes so that she could use the data to make flexible groups and inform future instruction.

Example Monitoring Charts

Below are two more examples of monitoring charts. The first is an example of a second

grade addition lesson. The second is an example created for a kindergarten counting lesson. Both are generic examples that could be used to monitor learning.
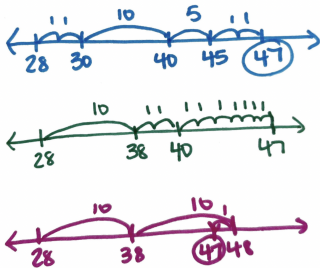
Second Grade Example

Mr. Kent is a second grade teacher. His class is focusing on using various addition strategies to solve word problems within 100. He created the following monitoring chart to keep track of the strategies his students are using and pre-plan questions to ask them (see Figure 2). Here is the problem students were solving: *Cara has 28 stickers. Marco has 19 more stickers than Cara. How many stickers does Marco have?*

Figure 2

Second Grade Addition Monitoring Chart

Addition Strategy	*Questions to Ask*	*Students*
Open Number Line	• Why did you decide to break down the numbers this way? • Is there another way you use the open number line to solve this problem?	
Make a Ten $28 + 19$ $27 + (1 + 19)$ $27 + 20 = 47$	• Why did you break apart ____ that way? • How did you know to break apart ____? • Is there another way you could have used the make a ten strategy to solve this problem?	
Friendly Numbers with Compensation $28 + 19$ $28 + 20 = 48$	• Why did you decide to change the __ to __?	

Take 1 extra away 47	• Why did you take/give back the ____? • Is there another way you could have used friendly numbers to help you solve this problem?	
Interprets Word Problem as Subtraction	• Tell me about the problem in your own words. • How do you know this problem shows subtraction? • What does it mean to subtract? add?	
Cannot Get Started	• Tell me about the problem in your own words. • Could you draw a picture or bar model to represent the problem?	

Kindergarten Example

Ms. Danvers is a Kindergarten teacher. Her class is focusing on developing their ability to count objects to 20. She wants to create a monitoring chart to track student progress during counting activities (see Figure 3).

Figure 3

Kindergarten Counting Monitoring Chart

Counting Concept	*Questions to Ask*	*Students*
Accurately counts to ____ demonstrating one-to-one correspondence and cardinality	• How do you know there are XX objects in the counting car? • Is there another way you could count the objects in the jar?	

Struggled with One-to-One Correspondence	• How do you know which objects you've counted? • What strategies have we learned to help us keep track of what we've counted? Let's look at the anchor chart.	
Struggled with Cardinality	• What was the last number you said when you counted?	
Struggled with the number sequence	• I see you skipped XX. Can you try counting them again? • What number comes after/before ___?	
Struggles with all areas of counting	• How can we figure out how many objects are in the jar? • What strategies have we learned to help us count? Let's look at the anchor chart.	

Benefits of the Monitoring Chart

There are many benefits to using a monitoring chart in mathematics lessons. Effective problem-based mathematics lessons require thoughtful planning, including anticipating student strategies and misconceptions and pre-planning questions. The monitoring chart described in the examples above serves as a tool that provides a space to begin planning an effective problem-based lesson by creating space for teachers to anticipate student thinking, plan questions, and even purposefully plan the explanation part of a problem-based lesson.

Anticipating Strategies

Anticipating strategies help teachers purposefully consider the mathematical development

of their students. These could be the strategies students might use to solve a problem or task. It could also be the errors or misconceptions they might see as they observe or monitor students while solving the problem. In the examples above, the teachers had to consider what strategies the students in their class might use to solve addition and multiplication problems. Sometimes the anticipated strategies can be found in the adopted curriculum such as iReady (2022). Another great resource for anticipating strategies are the Utah Math Core Guides found on the Utah Education Network website.

Pre-Planning Questions

It is important for teachers to pre-plan the questions they intend to ask during a math lesson. In fact, "posing purposeful questions" is one of NCTM's Effective Mathematics Teaching Practices (2014). Pre-planning helps to ensure that questions are open-ended and require higher level thinking from students. The monitoring chart supports pre-planning questions around the anticipated strategies and/or misconceptions students might have as they engage in problem solving. The monitoring chart is flexible in terms of the questioning framework teachers want to use. Smith, Bill, and Sherin (2019) use the assessing and advancing framework, in which teachers should include a blend of questions that help you understand the students' thinking (assessing) and questions that help you extend and/or deepen students' thinking (advancing). You could also use the questioning framework synthesized by NCTM (2014) in which teachers ask questions that: 1) gather information; 2) probe thinking; 3) make the mathematics visible; and 4) encourage reflection and justification. Teachers could also use Bloom's Taxonomy to guide their question planning to ensure that students are engaging in those higher levels of thinking like applying, analyzing, and evaluating. Regardless of the questioning framework you want to use, the monitoring chart provides that flexibility while also helping teachers to ask a variety of

questions to challenge students to reflect on and justify their mathematical thinking (NCTM, 2014).

Guiding Discussions

The monitoring chart also provides valuable information in order to guide whole class discussions where students share their strategies and solutions. Smith and Stein (2011) advocate for using the data collected in the monitoring chart to *select* which students will present during the whole class discussion, and to then *sequence* the order of the student presentations. It is important that the teacher purposefully sequence student presentations to align to the goal of the lesson. Most often, teachers will sequence strategies from the most concrete to the most abstract, but sometimes they may want to highlight a common misconception or a strategy used by the majority of students first. Regardless, teachers should help students make explicit connections among representations and strategies throughout the discussion. The monitoring chart is a valuable tool in facilitating productive whole class discussions of mathematics.

Inform Instruction

NCTM's Effective Mathematics Teaching Practices (2104) address the importance of eliciting and using evidence of student thinking, which expects teachers to use evidence of student thinking to continually assess their progress toward conceptual understanding. The monitoring chart is not only a great tool for guiding a lesson; it also acts as a way to keep track of evidence of a student's progress through the strategies they are using or the misconceptions they demonstrated while solving a problem. Without a mechanism for taking notes, teachers often lose track or are unsure about the progress students are making. Using the chart, teachers can make instructional decisions based on the solution strategies and misconceptions uncovered

during the lesson. Teachers can also use the chart to flexibly group students based on the way they are thinking about the mathematics concept or the errors they are making.

Opportunity for Self-Reflection about the Lesson

Self-reflection is essential to effective teaching. In fact, the Utah Effective Teaching Standards (USBE, 2013) outline that a teacher is expected to be a "reflective practitioner who uses evidence to continually evaluate and adapt practice to meet the needs of each learner." Utilizing a monitoring chart allows teachers to critically evaluate the effectiveness of their teaching practices and reflect on the progress students are making. The use of a monitoring chart to record student strategies or thinking allows a teacher to reflect on how students are solving problems. This can also lead to self-reflection on teaching and why students are or are not moving forward in their conceptual understanding, procedural fluency, or mathematical reasoning in regard to a particular concept.

Conclusion

The monitoring chart is a worthwhile practical tool in problem-based mathematics lessons. Using a monitoring chart has several advantages. The monitoring chart provides teachers with an opportunity to anticipate what students might do to solve a problem and the questions they could ask to support or advance students in their mathematical thinking. During a lesson the teacher is able to keep track of student practice, providing formative data to inform instruction. All of the benefits of the monitoring chart increase equity within the mathematics classroom. We encourage you to try using a monitoring chart in your next lesson and watch it transform the way you plan mathematics instruction and gather data.

References

Bieda, K.N., & Staples, M. (2020). Justification as an equity practice. *Mathematics Teacher: Learning & Teaching PK-12, 113*(2).

Moore, I. & Sallis, P. (1990). *Six Dinner Sid*. Aladdin.

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all.* NCTM.

Smith, M., Bill, V., & Sherin, M.G. (2019). *The 5 practices in practice: Successfully orchestrating mathematics discussions in your elementary classroom.* Corwin.

Smith, M.S. & Stein, M.K. (2011). *5 practices for orchestrating productive mathematics discussions*. NCTM.

Utah State Board of Education [USBE]. (2013). *Utah effective teaching standards and indicators.* https://www.schools.utah.gov/file/7313cfe5-5e68-41ef-9de4-03e5a9d395d8

Wiggins, G., & McTighe, T. (2005). *Understanding by design (2nd ed.)*. ASCD.

Wiliam, D. (2015). Assessment: A powerful focus for the improvement of mathematics instruction. In C. Suurtamm & A.R. McDuffie (Eds.), *Annual perspectives in mathematics education: Assessment to enhance teaching and learning*. NCTM.

Wisconsin Department of Public Instruction [Wisconsin DPI]. (2013). *Mathematics instruction: LESRA model*. https://dpi.wi.gov/math/instruction

How Learning Progressions and Learning Trajectories Can Inform Mathematics Teaching

Shannon Olson (Olson Educational Services LLC)

Recently I was working with a second grade teacher on planning for two-digit addition and subtraction. She found that while most of her students were accessing the content, a few students needed some extra scaffolding. To guide us, we consulted relevant first and second grade standards.

Standard 2.NBT.5 (a second grade standard) expects students to "Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction," and Standard 1.NBT.4 (an associated first grade standard) expects students to "Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" (Utah State Board of Education, 2016)

In both first and second grade, students are expected to add within 100 (as well as other parameters depending on the specific standards), but the first grade standard lays out some specific situations to begin with before students are expected to fluently add and subtract within 100 in second grade using a variety of strategies. We were able to plan interventions and supports using the first grade standard to scaffold the second grade standard by having students first add a two-digit number and a one-digit number and then add a two-digit number and a multiple of 10 before having them add two different two-digit numbers.

This is an example of using learning progressions to plan for instruction.

Learning progressions and learning trajectories are my favorite things to learn about when it comes to standards and content. So what are learning progressions and learning trajectories and how do we use them to plan mathematics instruction?

Learning progressions and learning trajectories inform the pathways in which students learn content. Sometimes the terms are used interchangeably, but depending on the source and context there are a few differences.

Karin Hess describes learning progressions and learning trajectories in the following way. "Learning progressions, progress maps, developmental continuums, and learning trajectories are all terms that have been used in the literature over the past decade. While many variations on the definition exist, the concept generally refers to research-based, descriptive continuums of how students develop and demonstrate deeper, broader, and more sophisticated understanding over time" (Hess, 2014).

Research-based, descriptive continuums exist based around both standards in mathematics and around research on how children develop understanding of concepts aside from standards defined by local, state, and national organizations.

Learning Progressions

Kim & Scoular state, "Learning progressions describe typical sequences of learning in specific areas or disciplines. A familiar parallel would be a curriculum that is designed to help a student learn about a particular subject over the course of several grades. … What's more, while learning standards describe what a student should have learned by a specific stage in their education, learning progressions focus on the building blocks that contribute to mastering a particular skill. This developmental approach maps the *progress* of a student through stages of increasing knowledge, skills, and understanding" (Kim & Scoular, 2017).

Standards in mathematics define what students are expected to know, do, and apply in mathematics. They address big ideas such as understanding operations and understanding place value as well as minute details including numerical parameters students are expected to work with in each grade level as well as some conceptualizations and strategies they should apply to the big ideas. Learning progressions help us know the pathways students may take to learn concepts both within and across grade levels.

Care & Kim argue, "It is not sufficient for a teacher to know only the curriculum being taught in her grade—she must understand what the students learned before, and what they will need to engage with after—in order to ensure deep learning. Learning progressions can describe the sequence of learning in a domain over many years for 'big ideas'" (Care & Kim, 2018).

As teachers come to know and deeply understand where students are coming from and where they are going, they are better able to plan targeted instruction based on grade level standards and student needs. They save time when they activate background knowledge of previous learning without reteaching entire concepts and when they go deeper within grade level concepts rather than unintentionally teaching concepts intended to be reserved for future grade levels.

Thinking about the example of addition and subtraction in first and second grade referenced above, second grade teachers can activate background knowledge by leveraging prior knowledge students have related to the first grade expectations as they move into second grade expectations. They can also know how far to take students and where to stop by becoming familiar with third grade expectations.

The standards in mathematics were designed with progressions at the forefront of all content. "The Standards in mathematics were built on progressions of topics across a number of

grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics" (The University of Arizona, 2013).

Learning Trajectories

So is learning trajectory just another name for learning progression? Not exactly. I like to think of learning progressions as the big ideas across grade levels and the way standards connect over time. Learning trajectories refer more to the pathways children take to actually learn the concepts in those progressions.

Confrey, Shah, & Maloney explain, "Although standards are essential in identifying what topics to teach and when to teach them, they offer little insight into *how* to teach those topics. Learning trajectories (LTs) are empirically grounded descriptions of how students' reasoning evolves from less to more sophisticated. They can provide deep insight into how to teach topics during a single grade as well as how topics develop and evolve across the grades.

LTs can inform teaching by contributing a variety of pedagogical and content-related insights and strategies. They support sequencing topic introduction and development. Because ideas evolve gradually and often rely on a careful introduction of new representations, operations, cases of numbers, structures, and definitions, LTs can help teachers avoid overwhelming students with prematurely formal concepts. They shed light on where an idea comes from and how it allows students to see certain situations differently. They support recognizing and leveraging diverse *student* ideas and shed light on student misconceptions and errors" (Confrey, Shah, & Maloney, 2022).

While learning progressions address *when* to teach concepts, learning trajectories address *how* to teach concepts by providing ways to sequence instruction and what to expect from student thinking. In addition, they offer instructional strategies that are developmentally

appropriate for students, such as introducing concepts concretely before moving to abstract procedures.

Furthermore, Confrey, Shah, & Maloney, "liken LTs to a climbing wall rather than a ladder. … A ladder implies that students proceed uniformly through strictly prerequisite levels. A climbing wall assumes climbers move upward from a variety of starting points through multiple paths. LT levels, when envisioned as handholds, footholds, and obstacles, make student thinking visible and predictable, though probabilistic" (Confrey, Shah, & Maloney, 2022).

What would a classroom look like in which children are able to scale the mathematical landscape in predictable, yet flexible ways based on their own development? What would it look like if teachers were aware of each handhold, foothold, and obstacle and could guide students as they traverse their pathways?
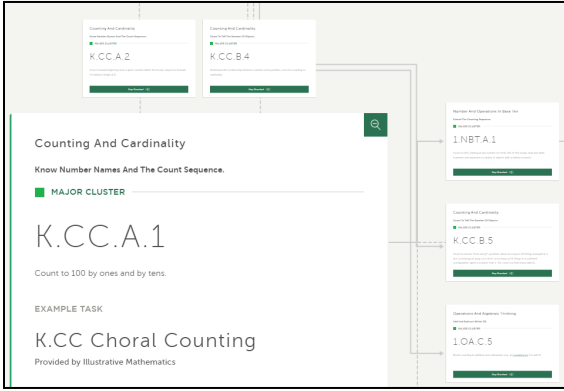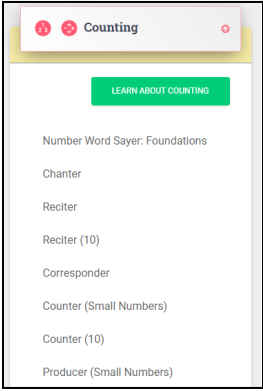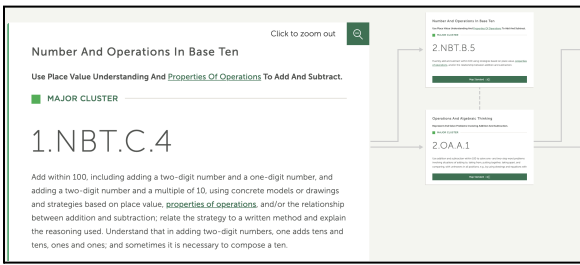
Clements & Sarama claim, "Children follow natural developmental progressions in learning. Curriculum research has revealed sequences of activities that are effective in guiding children through these levels of thinking. These developmental paths are the basis for the learning trajectories. …

Learning trajectories allow teachers to build the mathematics of children – the thinking of children as it develops naturally. So, we know that all the goals and activities are within the developmental capacities of children. We know that each level provides a natural developmental building block to the next level. Finally, we know that the activities provide the mathematical building blocks for school success" (Clements & Sarama, 2017/2019).

Examples of Learning Progressions and Learning Trajectories

Now that we know what learning progressions and learning trajectories are, let's look at some examples.

Examples of Learning Progressions and Learning Trajectories

Learning Progressions	Learning Trajectories
Counting Students count to 100 by tens and ones (K.CC.1) in kindergarten before they count to 120 starting at any number less than 120, reading and writing numerals and representing a number of objects with a written numeral (1.NBT.1) in first grade.	**Counting** Students may be chanters, then reciters, then corresponders, then counters of small numbers and counters of numbers up to ten before becoming producers of numbers. This may happen at any age, including before or after kindergarten.
Adding and Subtracting Students add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between	**Adding and Subtracting**

addition and subtraction (1.NBT.4) before they fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5).

https://achievethecore.org/coherence-map

(Student Achievement Partners, 2022)
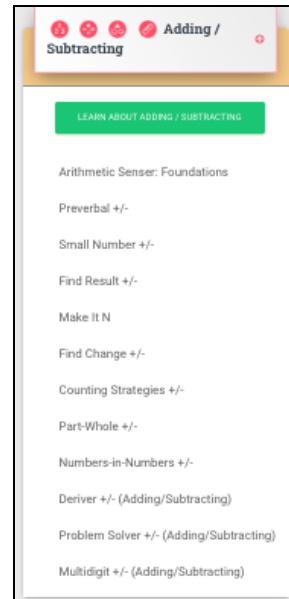
Before students add and subtract with multi-digit numbers, they progress through a number of stages in which they work with small numbers and solve various problem types.

https://www.learningtrajectories.org

(Clements & Sarama, 2017/2019)

Applications for Learning Progressions and Learning Trajectories

We know that learning progressions describe how children learn concepts over time and that the standards in mathematics are inherently designed with progressions in mind. In short, they tell us *when* students learn concepts. We know that learning trajectories help us navigate how children develop mathematical understanding. In short, they tell us *how* students learn concepts. How do we make sense of this and use learning progressions and learning trajectories together?

Applications for Learning Progressions and Learning Trajectories

Learning Progressions	Learning Trajectories
- Planning for whole class instruction - Planning for grade level instruction - Knowing the flow of big ideas across time	- Planning for differentiated instruction - Planning for skill-based instruction - Knowing the flow of developing individual skills

The second grade teacher from earlier used both learning progressions and learning trajectories to inform her instruction. She consulted learning progressions when she compared the second grade standard to the first grade standard. She found ways to scaffold the order in which students learn concepts and ways to activate prior knowledge. When working with individual students she used learning trajectories to know how to support diverse learning needs. Some students needed more practice with smaller numbers and others needed to build on counting strategies.

Final Thoughts on Learning Progressions and Learning Trajectories

Take a moment to think, What are learning progressions? What are learning trajectories? How are they similar? How are they different? How might you use them when planning for math instruction?

Learning progressions let us know where students are coming from and where they are going. They let us target grade level instruction in ways the standards intend for instruction to be targeted. Learning trajectories let us know how students develop understanding of mathematical concepts. They let us provide supports and direction for unique learners across our classrooms.

Resources on Learning Progressions and Learning Trajectories

There are a variety of resources available to support teachers in understanding and
implementing learning progressions and learning trajectories. Let's take a look at some of them.

Clements, D. H., & Sarama, J. (2017/2019). *Learning and teaching with learning
trajectories [LT]2*. Retrieved from Marsico Institute, Morgridge College of
Education, University of Denver. https://www.learningtrajectories.org/

Graham Fletcher. (2022). *Progressions videos*. GFletchy.
https://gfletchy.com/progression-videos/

Student Achievement Partners. (2022). *Coherence map*. Achievethecore.org.
https://achievethecore.org/coherence-map/

Student Achievement Partners. (2013). *Deep Dive into the Math Shifts.
Achievethecore.org.
https://achievethecore.org/page/400/deep-dive-into-the-math-shifts*

The University of Arizona. (2019). *Progressions documents for the Common Core Math
Standards*. http://ime.math.arizona.edu/progressions/

Utah State Board of Education. (2018). *Major Works*.
https://www.schools.utah.gov/curr/mathematics/core?mid=4514&tid=3

Utah State Board of Education. (2018). Utah Mathematics Core Guides.
https://www.schools.utah.gov/curr/mathematics/core?mid=4514&tid=2

References

Care, E. & Kim, H. (2018, March). *Learning progressions: Pathways for 21st century
teaching and learning.* Brookings.
https://www.brookings.edu/blog/education-plus-development/2018/03/27/learning-progressions-pathways-for-21st-century-teaching-and-learning

Clements, D. H., & Sarama, J. (2017/2019). *Learning and teaching with learning
trajectories [LT]2*. Retrieved from Marsico Institute, Morgridge College of
Education, University of Denver. https://www.learningtrajectories.org/

Clements, D. H., & Sarama, J. (2017/2019). *Learning and teaching with learning
trajectories [LT]2: Additional Resources & Frequently Asked Questions.*
https://www.learningtrajectories.org/pages/sub_page/29

Confrey, J., Shah, M., & Maloney, A. (2022). Learning Trajectories for Vertical
        Coherence, *Mathematics Teacher: Learning and Teaching PK-12*, *115*(2),
        90-103. https://pubs.nctm.org/view/journals/mtlt/115/2/article-p90.xml

Hess, K. (2014). *Learning progressions focus area.* Karin Hess, Ed.D.
        https://www.karin-hess.com/learning-progressions

Kim, H. & Scoular. C. (2017, April). *Learning progressions: Road maps for 21st-century
        students—and teachers.* Stanford Social Innovation Review, Stanford University.
        https://ssir.org/articles/entry/learning_progressions_for_students_and_teachers#

Student Achievement Partners. (2022). *Coherence map*. Achievethecore.org.
        https://achievethecore.org/coherence-map/

The University of Arizona. (2013). *Progressions Documents for the Common Core Math
        Standards.* http://ime.math.arizona.edu/progressions/

Utah State Board of Education. (2016). Utah Core State Standards for Mathematics.
        https://www.schools.utah.gov/curr/mathematics/core?mid=4514&tid=2

What's Going on with Critical Race Theory and Why Should a Math Teacher Care

Trevor Warburton (Utah Valley University)

While media attention to Critical Race Theory (CRT) in education has died down from its peak in early 2022, it is still a topic of contention in education circles and among education policy makers. The state of Utah recently conducted an audit looking for evidence of CRT in Utah schools (Cortez, 2022; Tanner, 2022). In the process, a lot of misinformation has been spread about what Critical Race Theory (CRT) is and how it might be connected to education. In this article I intend to explain what CRT is, what the uproar is all about, and why the topic around CRT should matter to mathematics teachers in Utah.

Origins of Critical Race Theory

Critical Race Theory was developed in the 1970s by a group of critical legal scholars who were looking for a way to better understand the racial implications of the US legal system. Prominent among these scholars were Derrick Bell, Alan Freeman, Kimberlé Crenshaw, Richard Delgado, Cheryl Harris, Mari Matsuda, and Patricia J. Williams. Specifically, this group desired to explain why the previous Civil Rights victories were not having the expected impact and to explain how apparently neutral laws could produce disproportional racial impacts (Delgado & Stefancic, 2017). While complex, in simplest terms CRT is a "race-concious" approach to understand societal inequality (Zamudio et al., 2011). Among the key concepts that were developed to form CRT are:

- Racism is endemic—racism is a common, ordinary experience for people of color in the United States.

- Colorblindness can lead to racist outcomes—colorblindness is a myth and choosing to not see race can mask existing racism.
- Analysis needs to be contextual and historical—many analysis focus only on the immediate facts and don't take into account the broader context and the (often) long history leading to the present moment.
- The experiential knowledge of people of color matters—frequently in legal, academic, and educational matters the perspectives, experiences, and knowledge of people of color are dismissed or overlooked.
- Interest convergence can explain Civil Rights gains—gains in Civil Rights may do more to serve the interests of white elites than they do for people of color (see example below).
- Identity is intersectional—everyone is an intersection of multiple identities including race, gender, sexual orientation, and others; being a Black woman is different than being a Black man and than being a white woman, and is not the same as the combination of the two.
- Racial (and other) oppressions must end—unlike most academic theories CRT contains an explicitly activist component.[1]

Critical race theorists then used these concepts to understand and explain the US legal system (Delgado & Stefancic, 2017; Ladson-Billings, 2009; Zamudio et al., 2011). For example, interest convergence (the idea that progress is made only when it is in the interest of those in power) has been used to explain the landmark *Brown v Board of Education* desegregation decision. The United States was facing intense pressure in the international community (and at home), because of the poor treatment that Black WWII veterans (and their families) received. Desegregation was seen as a way to improve international standing which would help in efforts to stop the spread of communism (Ladson-Billings, 2009). As another example, the ideology of colorblindness has been used to hide or dismiss the current reality of segregation. By some measures schools are as segregated today as they were before the desegregation decision (i.e. currently a higher percentage of students of color attend school with a high majority of other students of color; Zamudio et al., 2011). But since segregation based on race is illegal and current segregation is created in other ways, it receives minimal attention. Further, colorblindness is actively used to

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[1] These are necessarily simplified definitions of complex topics. For fuller definitions I highly recommend the readings cited in the references.

dismantle existing policies, such as affirmative action, that are meant to correct for existing racism (Zamudio et al., 2011).

Critical Race Theory in Education

Beginning in the mid-1990s Critical Race Theory was brought into the field of education by Gloria Ladson-Billings and William Tate and, since then, it has been used as a tool for *research* in education. Despite the growth of CRT in education research, it is far from a dominant or common tool of analysis and is typically only taught in PhD level courses as a research theory, if it is taught at all. However, CRT has been useful within education to understand how racism is produced and reproduced in schools. While CRT is not taught in teacher preparation programs and not taught in K-12 schools (see the recent audit conducted by the state of Utah; Cortez, 2022; Tanner, 2022[2]), it has had an impact both on teacher and K-12 education. Education research informed by CRT has helped to illustrate ways that the education system perpetuates racism through various means like tracking, school policies, location of classes (i.e. are the English Language Learners in a separate, worse part of the school), a Eurocentric curriculum, and teacher-centered pedagogies (Zamudio et al., 2011). While CRT has not been the only perspective to make these critiques, it has done so in a powerful way and with an important focus on race that is often missing in equity discussions.

Utah Context

It is also important to clarify some things that CRT is not. In May 2021 the Utah State Legislature passed S.R. 901 *Senate Resolution on Critical Race Theory in Public Education*. Specifically, this resolution,

> strongly recommends that . . . no curriculum or instructional materials in the state include the following concepts:

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[2] It is important to note that while the audit notes a very few "bad examples" none of them are examples of teaching CRT, in other words the theory itself was not taught.

- that one race is inherently superior or inferior to another race;
- that an individual should be discriminated against or receive adverse treatment because of the individual's race; or
- that an individual's moral character is determined by the individual's race.

The resolution also recommends that teacher training not include these concepts. While the senate resolution clearly identifies these concepts with Critical Race Theory, they are not in any way part of CRT. In response to this resolution the Utah State Board of Education drafted and passed a new Equity Board Rule (R277-328). The board rule makes no mention of CRT or the specific concepts listed in the resolution. It appears to be a standard explanation of equity within an educational context.

Relevance to Utah Teachers

Most teachers could reasonably look at the Utah senate resolution and the Utah equity board rule and come to the conclusion that all of the hubbub about CRT is much ado about nothing. Most teachers don't do any of the things recommended against in the resolution. However, there is more to consider. Christopher Rufo, one of the primary national leaders behind the anti-CRT backlash, has publicly stated that he aims to eliminate a number of common school practices that are used to promote equity and the health and well-being of students (Meckler & Dawsey, 2021). He has also stated that in the process he will redefine CRT to mean what he wants it to mean. In other words, his purpose is to use intentional misinformation to further his inequitable educational vision. His push has resulted in a number of anti-CRT bills across the country and additional bills that ban certain topics or eliminate requirements to teach about civil rights and women's suffrage.

For example, a Utah State Board of Education representative proposed an amendment to the Equity Board Rule that would ban a wide range of topics from Utah schools including Social-Emotional Learning, equity, privilege, empathy, funds of knowledge, and multiculturalism

among many others (Fox 13 Now, 2021, reference includes a link to the full amendment). Importantly, even though the amendment failed, the anti-CRT push has created an environment where educators are fearful of addressing important topics like race and racism in schools. It has also increased a spirit of distrust of educators and the education system, among some parents and especially among politicians. This distrust has led to the proposed curriculum transparency bills that were defeated in the last legislative session (but may return) and are also a goal of Christopher Rufo and the anti-CRT push (Gaudiano, 2022).

Relevance to Mathematics Teachers

It can be easy as mathematics teachers to see ourselves as disconnected from the anti-CRT efforts, especially in their initial focus on teaching about race and racism. However, with the attempts to erode public trust in the education system and to exert greater legislative control over what is taught, through curriculum transparency bills it is clear that every public educator will be impacted by these efforts. There are also aspects of these efforts that specifically impact mathematics educators. Recently, Florida rejected dozens of mathematics textbooks for containing "prohibited topics" in connection to Florida's anti-CRT push which has been similar to Utah's. Few, if any, of the textbooks had any mention of race (and it shouldn't be a problem even if they did), instead they included questions asking students about their emotions or promoting grit and perseverance. The rejected textbooks included books from major publishers (Gross, 2022). As a result, publishers may be less willing to publish even minimally inclusive curricula out of fear that it will be targeted or simply not purchased. .

Perhaps more importantly is how this will affect our own efforts to make our courses more inclusive. We know that our students benefit when we highlight mathematicians from various underrepresented groups, including racial and ethnic minorities (Gutiérrez, 2012; 2018).

We know that our students do better when we create a more inclusive classroom environment, especially in terms of race, gender, and sexuality (Kokka, 2018; Rubel, 2017). We know that students can thrive mathematically when meaningful and relevant connections are made to their cultures (Dee & Penner, 2017; Kokka, 2018; Rubel, 2017). It is not yet clear what backlash mathematics teachers may face when they make these efforts, however, what is clear is that, for many of us, making these efforts was already a step into discomfort and suddenly that step appears more dangerous, possibly making us hesitate to make our mathematics classes inclusive. The anti-CRT backlash has already had a chilling effect on Utah teachers' willingness to address challenging topics such as racism, with teachers saying, "'I am afraid. I don't know what I can and can't say so I just don't say anything'" (Cortez, para 11). That fear of taking action and teaching to support students of color will be the biggest impact of these anti-CRT efforts. These efforts and the faux "transparency" efforts may have the intended effect of pushing committed, equity minded educators out of the classroom entirely, exacerbating the already problematic teacher shortage in Utah.

Conclusion

The recent panic over Critical Race Theory in schools isn't really about Critical Race Theory. Instead it is about who school is for, who should decide what students can and can't learn, and what teachers can and can't do in the classroom, including how mathematics teachers make class inclusive and connect to a wide variety of cultures and histories. It is part of an anti-equity movement that addresses not only race, but other equity efforts in schools and across society. Finally, Critical Race Theory could be a powerful tool to help mathematics teachers and our students to learn more about racism that happens regularly in our classrooms, our schools, and society. CRT is powerful tool to support students of color make sense of their racialized

experiences both in and out of school, including in the mathematics classroom (Jett, Yeh, &

Zavala, 2022). To this end mathematics teachers would benefit from having CRT take a more

prominent role in teacher preparation and professional development. We need more CRT

influencing our mathematics teaching and school policies not less.

Resources to Learn More

On Critical Race Theory in General

- Book *Critical Race Theory: An Introduction* by Richard Delgado & Jean Stefancic
- Podcast *Intersectionality Matters* by Kimberlé Crenshaw
- Podcast *Scene on Radio: Season 2 Seeing White* by John Biewin

On Critical Race Theory in Education

- Book *Critical Race Theory in Education: All God's Children Got A Song* by Adrienne D. Dixson & Celia K. Rousseau

On Critical Race Theory in Mathematics Education
- Article From *Argumentation to Truth-Telling: Critical Race Theory in Mathematics Teacher Education* by Christopher C. Jett, Cathery Yeh, Maria del Rosario Zavala in Mathematics Teacher Educator
- Book *Critical Race Theory in Mathematics Education* by Julius Davis and Christopher C. Jett
- Position Statement *The Mo(ve)ment to Prioritize Antiracist Mathematics: Planning for This and Every School Year* by Maria del Rosario Zavala, Ma Bernadette Andres-Salgarino, Zandra de Araujo, Amber G. Candela, Gladys Krause, Nichole Lindgren, and Erin Sylves
https://www.todos-math.org/assets/images/The%20Movement%20to%20Prioritize%20Antiracist%20Mathematics%20final%203.0_v6.pdf

References

Cortez, M. (2022, October 20). What did auditors find when they looked at what's being taught in Utah public schools? *Deseret News.*

https://www.deseret.com/utah/2022/10/20/23411738/audit-public-school-curriculum-teachers-utah-legislature

Dee, T.S., & E.K. Penner, (2017). The causal effects of cultural relevance: Evidence from an ethnic studies curriculum. *American Educational Research Journal, 54*(1), 127-166.

Delgado, R. & J. Stefancic, (2017). *Critical Race Theory: An Introduction* (3rd edition). New York University Press.

Fox 13 Now (2021, August 5). Utah Board of Education receives hundreds of comments ahead of equity teaching decision. *Fox 13 Now.*

https://www.fox13now.com/news/local-news/hundreds-of-comments-received-ahead-of-meeting-on-education-equity-in-schools-rule

Gaudiano, N. (2022, March 8). A key conservative instigator of the critical race theory controversies says he now wants to 'bait' the left into fights over Republican-led 'transparency' efforts in schools. *Business Insider.*

https://www.businessinsider.com/critical-race-theory-curriculum-transparency-christopher-rufo-conservative-school-education-2022-3

Gross, T. (2022, April 28). How social-emotional learning became a target for Ron DeSantis and conservatives. *National Public Radio.*

https://www.npr.org/2022/04/28/1095042273/ron-desantis-florida-textbooks-social-emotional-learning

Gutiérrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education?. In B. HerbelEisenmann, J. Choppin, D. Wagner, & D. Pimm, (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (17-33). Springer.

Gutiérrez, R. (2018). The need to rehumanize mathematics. In I. Goffney, R. Gutiérrez, & M. Boston (Eds.), *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students* (1-10). National Council of Teachers of Mathematics.

Jett, C.C. Yeh, C., Zavala, M.R. (2022). From argumentation to truth-telling: Critical Race Theory in mathematics teacher education. *Mathematics Teacher Educator, 10*(3), 223-230.

Kokka, K. (2018). Healing-informed social justice mathematics: Promoting students' sociopolitical consciousness and well-being in mathematics class. *Urban Education,* 1-31.

Ladson-Billings, G. (2009). Just what is Critical Race Theory and what's it doing in a nice field like education? In E. Taylor, D .Gillborn, & G. Ladson-Billings (Eds.) *Foundations of Critical Race Theory in Education.* Routledge.

Meckler, L. & J. Dawsey (2021, June 21). Republicans, spurred by an unlikely figure, see political promise in targeting crtical race theory. *The Washington Post.* https://www.washingtonpost.com/education/2021/06/19/critical-race-theory-rufo-republicans/

Rubel, L.H. (2017). Equity-directed instructional practices: Beyond the dominant perspective. *Journal of Urban Mathematics Education, 10*(2), 66-105.

Tanner, C. (2022, October 21). Utah auditors see no evidence that critical race theory is widely

taught — but found a few 'bad examples'. *Salt Lake Tribune.*

*https://www.sltrib.com/news/education/2022/10/21/utah-auditors-see-no-evidence/?utm_*

source=Salt+Lake+Tribune&utm_campaign=489bfa109c-EMAIL_CAMPAIGN_2022_1

0_20_07_10&utm_medium=email&utm_term=0_dc2415ff28-489bfa109c-45895737

Zamudio, M.M., C. Russell, F.A. Rios, & J.L. Bridgeman (2011). *Critical Race Theory Matters:*

Education and Ideology. Routledge.

What I Do Not Like in My Math Teacher

Dr. Kyriakos Petakos (Advanced School of Tourism Education in Rhodes, Greece)

Abstract

A question that really applies not only to students but also to all of us who have been students. There were teachers we were fond of, teachers we did not like. In the latter there is always a "why" that hovers over our memories. This why becomes stronger when it comes to mathematics. I do not aspire to answer it in a single article, but I at least try.

Introduction

Mathematics teaching attracts the attention of many people nowadays. Let us not forget that mathematics is one of the most difficult subjects taught in a school curriculum. The dipole student-teacher dynamically appears in the scene.

I do possess the adamant conviction that a student plays a pivotal role in the learning process. In the dipole mentioned above, I do not make an effort to emphasize the former and underplay the importance of the latter. I just want to provide students with the opportunity to demonstrate their own feelings, no matter how dormant they might have been till that moment. Referring to this sentimental plight, I need to add here what Thomas Mgonja articulates in his own way "students' voices seem to be lost in the shadows."(Mgonja, 2020, p.50).

Regardless of the teaching method used and the philosophy from which our teaching performance emanates, we, mathematics teachers, believe or should believe in the gravity of language. My classroom atmosphere is imbedded in the sociocultural context, as Vygotsky

magnificently depicts it in "language the very means by which reflection and elaboration of experience takes place, is a highly personal and at the same time a profoundly social human process" (Vygotsky, 1978, p.126).

The question posed has a negative nuance. The words "do not like" underpin this negativity. But I wish I saw students' responses, their written voices to such a question in an effort to get their subliminal message on mathematics teachers' demeanors. I have the feeling it will contribute to ameliorating not only the student-teacher relationship but also the mathematics lesson itself.

A student's confession.

I am using the word "confess" since I liken the registration of students' voices to analyzing mathematical souls. I cannot just teach based on professional criteria, avoiding at the same time sentimentality. Sentimentality in the sense that a student's personal approach to mathematics as well as my own approach will have to interchange with one another. We are not machines, we are human beings who are trying to get closer during the teaching of mathematics. Therefore the choice of a student that will help me visualize options on the teaching of mathematics is pivotal.

The student who provides the following answer has been watched by me on several occasions. I had him only at the beginning of a nonmathematical course, whose final grade was not provided by me. I discussed with him several times staying faithful to the Vygotskian doctrine that learning is a sociocultural process. I needed to make myself sure, to the extent that such a process is feasible, that I will get a meaty answer. There are students whose scholar achievements are better than his. Nevertheless, following my gut, I chose to collaborate with him

bearing in mind "I always preferred failure in a worthy effort to inaction for fear of failure." (Clinton, 2005, p.464).

I made it clear to him that by the math teacher in the question I referred to the general figure of a math teacher, who was recorded in his memory during middle high and high school. I highlighted some parts of his response that are presented below in italics.

I never had a teacher who created a positive environment towards the silly mistakes.

It is really important to mull over this remark. Our attitude, the mathematics teachers' attitude towards the "silly" mistakes. It probably requires a definition of what a silly mistake is, but I skip it trying to embellish a situation like this. In my own class, I try to repeat the mathematical material, in which the mistake has taken place, no matter if I stay back in covering the material according to the already given curriculum. How deep you can delve into past mathematical concepts is of course a personal choice and depends on the general progress of the class. But surely the characterization "silly" is absent in my lecture and I try to fill the gap between now and that time in a rather effective way. Generally, the effort to refresh memories bears its fruits regardless of the procrastination in covering the teaching material. His following phrase also justifies my comments above.

But always their highest priority was to finish the teaching material according to the program and not the essential understanding of mathematical concepts that every person should master when finishing high school.

We find ourselves in the everyday teaching practice on the horns of a dilemma. Should we go at a faster rate to be consistent with the curriculum or should we insist on some parts of it

that triggers students' questions at the cost of staying back? Following my gut, I decided to stick with the latter.

The blackboard. The worst feeling that someone might have had then, but unfortunately even today, has been to make a mistake in a calculation in front of everybody. In these cases, the teacher always did not help, on the contrary, she mustered all her forces to make the unfortunate student's mistake an opportunity for not understanding previously taught material.

The blackboard. At least, encouraging that students are often called to the blackboard. It sustains the modern pedagogical conviction that the student is not a passive learner, but a dynamic part of the lesson. Unfortunately, it is accompanied by a negative nuance. Rejection on the blackboard due to a mistake. This really rings a bell. We can employ the blackboard and whatever is written on it as an opportunity to a wider participation on behalf of the students. Not understanding a previous chapter should not be attributed to a student's incomplete coverage. We can bridge the gap by resorting to some solved problems related to that chapter and encourage the student by saying she provided the opportunity.

Let us liberate our students from the fear of the wrong answer. Language, the dynamic component of Vygotsky's theory, will play its role. In such cases, I inaugurate a dialogue, with as many students involved as possible. Is in this way the blackboard transformed into a smooth pathway for the learning process? I leave it to your judgment.

Conclusions

Silly mistakes, wrong answers, blackboard, covering the material in time, and the concomitant feeling that might be triggered to a student. We, mathematics teachers, are facing situations like that on an everyday basis. Let me restate here what Mognja (2020) uttered in his

article. We should not let students' voices go down the drain. Every voice has something to say, transferring with it vestiges from the sociocultural environment, to which it belongs. Let us make appropriate use of all these voices in an effort to improve our teaching performance. There is a reciprocity in roles, teacher's vocabulary becomes student's vocabulary and vice versa. This interchange may be proven fruitful in the classroom environment.

References

Clinton, W.J. (2005). *My Life, THE PRESIDENTIAL YEARS*. Volume I, VINTAGE BOOKS.

Mgonja, T. (2020). The Lost Voices in Mathematics Teaching. *Utah Mathematics Teacher* 13, 50- 62

Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Harvard University Press.

Book Review of *Choosing to See* by Kyndall Brown and Pamela Seda

Camille Lund (Weber School District)

Seda, P., & Brown, K. (2021). Choosing to see: A framework for equity in the math classroom. Dave Burgess Consulting, Incorporated.

Choosing to See comes in education's time of need, addressing a critical topic many would prefer to keep silent: racial equity in mathematics classrooms. Kyndall Brown and Pamela Seda reflect on their over 65 years of combined mathematics teaching experience to provide teachers with valuable insights and actionable strategies to create equity-centered mathematics instruction, with vignettes from their own teaching to illustrate their points in relatable ways.

Choosing to See focuses on Brown and Seda's ICUCARE (pronounced "I See You Care") framework: **I**nclude others as experts, be **C**ritically conscious, **U**nderstand your students, use **C**ulturally relevant curricula, **A**ssess, activate, and build on prior knowledge, **R**elease control, and **E**xpect more. This framework is "about choosing to see and caring enough about what you see to act" (p. 11). While the authors draw from their experiences as secondary mathematics educators, this book would be useful to any K-12 mathematics educator or team of educators. *Choosing to See* is a quick read offering short, structured chapters with activities that can immediately be implemented in the classroom. Additionally, each chapter includes space at the end to take notes. All activities mentioned in the main book text are included in detail in the appendices, adding to the book's easy-to-use and accessible nature.

As I read this book, I found activities that I could easily implement in my classroom the very next day, my favorite being "Find Someone Who" from chapter one. In this activity, the teacher prepares a set of problems for the students. Students walk around the classroom and pair up with partners who can answer a problem from the worksheet. They continue to walk around the room and pair up with partners until all the problems are complete. Because everyone is asking for and offering help, this activity provides an opportunity for students to take ownership of their learning and build confidence in their math-solving abilities. "Find Someone Who" helps students see themselves and their classmates as experts, whether or not they have been labeled as "smart." This activity, along with the may others mentioned in this book, helps build an equitable classroom where all voices are valued.

I find the most praiseworth attribute of *Choosing to See* to be its straightforward approach to discussing the role of racial structures in mathematics education. Brown and Seda use no uncertain terms to discuss such ideas as the damaging role of colorblind ideology, the effect of stereotypes, and the history of racist polities in the United States that negatively impact the education of students of color. They boldly state that "those who fail to be critically conscious or choose not to engage in culturally relevant teaching practices will continue to perpetuate inequity across racial and socioeconomic lines, whether it be consciously or unconsciously" (p. 17). But the authors offer ways for teachers to begin addressing these problematic tendencies and to become more culturally relevant teachers who honor their students' diversity and actively work to serve their marginalized students. In this way, *Choosing to See* stands out among other books for educators. Rather than simply addressing academics, the authors place an emphasis equity and the role it plays on academic success.

Perhaps the most valuable idea discussed in *Choosing to See* is the reality that mathematics has a culture. Brown and Seda discuss how societal ideas about who does math and who is good at math influence students' self-concepts and, in turn, their academic success. For example, when African American students are exposed to stereotypes suggesting that African Americans perform more poorly on academic tests than White students, they are more likely to perform more poorly than White students on such tests regardless of ability. Brown and Seda do a remarkable job of articulating ubiquitous equity issues to teachers who may have never seen math education in this light before. In this way, *Choosing to See* could be instrumental in helping teachers shift toward more equitable teaching practices.

There are only minor issues concerning the production of this book, such as out-of-date links. Most of the ideas are supported by research noted in the footnotes, but some information is based on anecdotes, and should be considered accordingly. *Choosing to See* is full of useful ideas, but the authors do not go deep into any of them. Therefore, this book would be best implemented as a starting point. Teachers who are looking to transform their classrooms into equitable learning spaces will need to supplement this surface-level read with other texts.

Choosing to See both encourages educators to consider how they can teach more equitably and provides tools for teachers to begin doing so immediately. It is friendly and light, yet direct and bold. We need more books like this.

Book Review of *Choosing to See* by Kyndall Brown and Pamela Seda

Molly Basham (Utah State University)

As is the case in most academic content areas in the wake of the COVID 19 pandemic, issues of equity are abound in mathematics education. Many of these issues were evident long before school closures highlighted inequities in student opportunity across socioeconomic and racial lines. However, those disparities were amplified while teachers navigated these past few years of unprecedented shifts in pedagogy and student access to instruction.

In their 2021 book, *Choosing to See: A Framework for Equity in the Math Classroom*, authors Pamela Seda and Kyndall Brown build on their collective work in equity and culturally responsive pedagogy in mathematics. They provide teachers and educational leaders with a framework of actionable changes that they can make in their practice to build more inclusive mathematics communities in their classrooms and schools. They situate their recommendations within personal stories and reflections, instructional and reflective activities for teachers, and relevant research in mathematics education to provide a high quality professional learning tool for educators looking to create change in the wake of the turbulence of the last several school years.

The book is organized around an equity framework that includes seven principles. Seda and Brown refer to their recommendations as the ICUCARE Framework (pronounced "I See You

Care"). Each chapter in the text focuses on one component of the framework that builds the

ICUCARE acronym. The chapters are outlined as follows: **I**nclude others as experts; Be

Critically conscious; **U**nderstand your students; Use **C**ulturally relevant curricula; **A**ssess,

activate, and build on prior knowledge; **R**elease control; and **E**xpect more. The text also includes

multiple appendices with resources that accompany the activities recommended throughout the

chapters.

Interwoven Themes Throughout the ICUCARE Framework

Though the authors structure the text around the components of their framework, several

themes are woven throughout their recommendations. The authors acknowledge in their

conclusion section that there are themes that overlap within each element of the framework, but

they encourage teachers to engage in these shifts of practice one at a time, and to reflect on the

experience as they try to change their pedagogy to be more equitable for diverse learners.

Throughout the book, Seda and Brown repeatedly address: (1) the value of centering and

elevating student voices in the math classroom, (2) focusing on students' assets, rather than their

deficits, and (3) utilizing culturally relevant and responsive pedagogy within the math classroom.

The aforementioned themes are interwoven throughout the chapters in a way that allows

educators to deeply examine their beliefs and biases that impact their students and make changes

to their instructional practices to work towards a more inclusive classroom.

Centering Student Voices

Throughout multiple chapters in the book, the authors emphasize the importance of

centering student voices and letting them build ownership over their learning. In chapter one, the

authors address the value of including others as experts in the classroom. They encourage

teachers to elevate students' expertise in the classroom rather than centering themselves as the

only expert on content. Similarly, in chapter six, the authors address releasing control within the classroom, deviating from a traditional teacher-centered pedagogy to one that lifts students up as valuable contributors to their learning comm. In this chapter, the authors also talk about raising up the voices of students as teachers move toward practices that are less teacher centered and more student centered. Chapters two, three, and four also include the importance of making classroom content relevant to students and giving them a voice in their own instruction as the authors urge educators to be critically conscious, to understand their students well, and to use culturally relevant curricula. All of these recommendations include elements of creating more student centered classrooms and providing students voice and choice within their own learning.

Asset-Based Mathematics

Another theme that is emphasized throughout Seda and Brown's work is the value of using asset-based practices in mathematics classrooms. In chapter one, as the authors encourage teachers to uplift students as experts, they address that all students come to the classroom bringing strengths and assets that are valuable and worthy of highlighting in the classroom. This same point comes up again in chapter three as the authors focus on the importance of understanding students deeply on both a personal and academic level. They advocate for educators to learn where their students strengths and interests lie so that these strengths can be leveraged during instruction. The value of asset-focused mathematics also comes to light in chapters five and seven as the authors discuss the importance of using assessment to activate and build on prior knowledge and encourage teachers to hold high expectations for all students. Both of these chapters get at the constructivist notion that students are the builders of their knowledge. Seda and Brown use the analogy that students are not empty banks into which teachers need to

deposit knowledge, but rather they come to classrooms with unique strengths and experiences that can be leveraged as the building blocks for future learning.

Asset-based mathematics is also deeply interwoven in the recommendations in chapter five, which focuses on assessing, activating, and building on students' prior knowledge. This section provides educators with tangible ways to focus on student assets, rather than deficits. Seda and Brown address the equity issues inherent in the traditional focus that teachers have on student deficits. They critique the practice of using assessment data strictly for the purpose of labeling and grouping students rather than using pre-assessment data as a jumping off point for helping students create new knowledge. The authors lean back on the work of Zaretta Hammond and encourage teachers to utilize the funds of knowledge that all learners bring to the classroom. They call out the uncomfortable fact that teachers are most likely to teach in a way that is similar to how they were taught, which privileges students of similar backgrounds to their teachers and marginalizes students of different cultural, ethnic, and socioeconomic backgrounds. They do however, provide teachers with suggestions for recognizing and valuing the prior knowledge that students bring to class. They encourage teachers to bring in familiar cultural references that help students relate to math concepts and see how math is relevant to their everyday lives and those of their families. The authors also stress the importance of utilizing low-floor, high-ceiling math tasks that allow all students at all skill levels to access the mathematical content and showcase their assets, rather than using traditional pedagogical approaches that leave some students out of the discussion. They encourage teachers to help students learn to self-assess their understanding so that they know how to validate their thinking and advocate for themselves in the classroom and beyond.

Culturally Relevant Pedagogy

Though Seda and Brown focus on using culturally relevant curricula specifically in chapter 4, they weave throughout the book invaluable recommendations that focus teachers' efforts toward using culturally relevant pedagogy in their classrooms. They cite work from key actors in this field like Gloria Ladson-Billings throughout the text as they discuss changes that teachers can make to be more inclusive of diverse learners in their mathematics classrooms. The themes previously discussed above both highlight critical components of culturally relevant pedagogy in which teachers can engage to create change within their classrooms. The resources provided by the authors in the book's appendices also provide tools for teachers to both analyze and improve their practices to be more culturally responsive.

Conclusion

Seda and Brown's 2021 book, *Learning to See: A Framework for Equity in the Math Classroom* is a powerful tool to help teacher's reflect on and change their instructional practices to be more inclusive for all learners. The book is easy and enjoyable to read and the authors provide reflective questions and strategies that teachers can use to guide their learning independently or as part of professional learning communities. Seda and Brown's use of personal narratives and reflections provides a sense of vulnerability that makes it easy for the reader to connect with the authors' experiences and see themselves within the text.

The authors provide powerful opportunities for teachers to look at their own practices and create change without laying blame or suggesting that teachers should feel shame if their current practices are not as equitable as they could be. However, they are not shy when it comes to calling out traditional practices that are inequitable and potentially damaging to students, particularly students of traditionally marginalized groups. The work that Seda and Brown are

encouraging teachers to undergo is bound to make some educators uncomfortable. They are advocating for change, and change does not come easy in education. This work requires deep reflection and self-evaluation around topics that are often considered taboo. However, the work is critical and educators need to step outside their comfort zones and analyze their beliefs and pedagogy to begin making shifts that create a more equitable and inclusive environment for diverse students.